Ay123

Fall 2007

STELLAR STRUCTURE AND EVOLUTION

Problem Set 1

Solutions due Wednesday, October 10th 2007

1. (10 pts) Assume a star obeys a linear density model so that

$$\rho(r) = \rho_c(1 - r\,/\,R),$$

where ρ_c is the central density and R is the radius of the star.

   a. Find an expression for the central density in terms of R and the mass M of the star.
   b. Use the equation of hydrostatic equilibrium and zero boundary conditions to find the pressure as a function of radius. Your answer will be in the form $p(r) = p_c \times f(r/R)$, where $f(x)$ is a function you will determine. What is the dependence of the central pressure p_c in terms of M and R?. Express p_c numerically with M and R in solar units.
   c. Assuming the equation of state for an ideal gas, what is the central temperature T_c?
   d. Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass (expressed in units of $M_\odot$). At what mass does the radiation pressure become comparable to the ideal gas pressure?
   e. Write down an explicit expression for the total gravitational potential energy of this toy star, and verify that the virial theorem is exactly satisfied. Be sure to discuss matter with a general equation of state, not just an ideal monatomic nonrelativistic gas.

2. (5 pts) The angular radius of the Sun is 16 arcmin and the radiant flux received at the top of the Earth's atmosphere (a quantity referred to as the *solar constant*) is 1.388 10^3 W m^{-2}. Using these data alone, calculate the effective temperature of the Sun.

3. *The Kelvin-Helmholtz timescale:* (10 pts) Use your results from Problem 1 to estimate the time it would take a large solar-mass proto-galactic cloud to contract to a radius $R_\odot$ assuming nuclear reactions had not yet ignited, and that the star cooled via emission of blackbody radiation from its surface at a temperature $T_{\rm eff} = 2500$ K. Now sketch on an HR diagram two curves: (i) the zero-age main sequence; (ii) the track of a contracting $M_\odot$ proto-galactic cloud.

4. (10 pts) Use the virial theorem to estimate a typical value for the speed of sound deep inside a star of mass M and radius R. Using this estimate, show that the time required for sound to cross the star is $P \sim (G\bar{\rho})^{-1/2}$. Hence, show that, for the fundamental p-mode of a star, the pulsation period takes a value of order 1 hr. Given that the luminosity L of a massive hydrogen-burning star varies with mass

roughly as $L \propto M^3$, show that, at fixed effective temperature, the fundamental pulsation period of such stars scales as $P \propto L^{7/12}$.

5. (10 pts) Sketch a plot of the stellar-remnant (i.e., white dwarf, neutron star, black hole) mass as a function of the initial stellar mass. Assuming stars form with a Salpeter initial mass function—i.e., that the number of stars in the mass interval $M \rightarrow M + dM$ is $\xi(M)dM$, with $\xi(M) \propto M^{-2.35}$, estimate the fraction of the mass of a stellar population that has been returned to the interstellar medium 10 Gyr after this population was formed. Comment briefly on the result.

6. (5 pts) Prove that if the orbital plane of binaries are oriented randomly with respect to the plane of the sky, that the average value of $sin^3 i$ is 0.59. How has this result been useful in calibrating the relationship between stellar mass and luminosity?

7. (10 pts) An eclipsing-binary system has a parallax of 0.1 arcsec, and for the moment we assume that this measurement is extremely accurate. It consists of two solar-mass stars identical to the Sun with a semi-major axis of 500 $R_\odot$. The period is very accurately known.
   a. What is the angular size of each of the stars and of the semi-major axis? If you can measure angles on the sky with a 1σ rms accuracy of 0.01 arcsec, what is the percentage accuracy of the measurement of the semi-major axis and of the radius of each star?
   b. Assume that the flux as a function of wavelength is given by a Planck function (i.e., it is a blackbody) with effective temperature $T_{\rm eff} = 5800$ K. Assume that we have measurements of the flux ratio between $\log(\nu) = 14.0$ and 15.0 (where ν is given in Hz) that have an accuracy of 10%. With what precision can we determine the $T_{\rm eff}$ of the stars from these measurements?
   c. What is the uncertainty in the mass of the system if the uncertainty in the parallax is taken as given in part a, 0.01 arcsec?
   d. What is the luminosity of each star calculated from the $T_{\rm eff}$ and the R, and what is the uncertainty in the calculated luminosity of each star in this binary?
   e. We can measure the apparent flux at the Earth from this star to an accuracy of 5%. Can we derive a more accurate luminosity and/or a more accurate radius from this observation than from relying on the measured parallax and the $T_{\rm eff}$ determined in part d?