

Ay123

Fall 2007

STELLAR STRUCTURE AND EVOLUTION

Problem Set 3

Solutions due Wednesday, October 24th 2007

1. (10pts) Consider a family of stars in which the opacity is dominated by Thompson scattering by electrons, and in which nuclear energy is generated by the CNO cycle. Use your lecture notes to determine the density and temperature dependencies of opacity and energy generation in such a case. In analogy with the homology relations we derived in class, for this family of stars, find the relation between radius and mass, and between luminosity and mass. Locate this population on the Hertzsprung-Russell diagram.
2. (10pts) Use the Saha equation for hydrogen to plot the ionization fraction y as function of temperature T for a density of $\rho = 10^{-10} \text{ g cm}^{-3}$. Calculate the adiabatic gradient $\Gamma_1 = (d \ln P / d \ln \rho)_{ad}$ as function of temperature for the same value of ρ . Make sure the temperature range of your plot covers both ionized and non-ionized states.
3. (10 pts) Do Exercise 1 in Chapter 3 of Hansen, Kawaler & Trimble. You should be able to explain or derive the degeneracy factors (statistical weights) for the ground-state helium atom and ions rather than copy them out of Allen as suggested.
- 4 (10 pts) Chandrasekhar mass via polytropes:
Show that an electrically-neutral gas, consisting of positive ions and fully relativistic and completely degenerate electrons, where the positive ions contribute negligibly to the pressure, obeys a polytropic equation of state with index $n=3$. Hence, using the equations we derived in class for the mass of a star obeying a polytropic equation of state, derive the mass of a fully-relativistic completely-degenerate white dwarf containing no hydrogen.
- 5 (10 pts) State of the Matter
 - a. Use your favorite computer program to draw a "State of the Matter" diagram as discussed in class. Make assumptions, where necessary, for the various coefficients and provide on a separate paper (or on the diagram if you can), the equations that define the lines. Your equations should only contain physical constants (i.e. m_e , m_p , \hbar , c , k_B etc).
 - b. Show that, to within in an order of magnitude, setting the degenerate energy density equal to the electrostatic energy density gives the same density as taking one proton per Bohr radius cube. Both calculations should only contain physical constants (do not substitute numbers).

c. Look up the density of liquid hydrogen, liquid helium, water and iron. How well do they satisfy the "same volume per atom" statement? What is the representative value of the atomic volume and what is the (cube root of that) typical atomic separation?