Ay123

Fall 2007

STELLAR STRUCTURE AND EVOLUTION

Problem Set 4

Due Wednesday, October 31, 2006

Please take any specific nuclear reaction rates needed from the Caughlan-Fowler (CN1988) nuclear reaction rates available at **http://www.phy.ornl.gov/astrophysics/ data/data.html**. In the notation used there to give the reaction rates as analytic functions, T923 = $T_9^{2/3}$, T913 is $T_9^{1/3}$. In particular, take the reaction rates needed for problem 4 from this reference.

When the energy generation rate for a process ($p+p$ or CNO) is required, use the expressions given in HKT.

1. We have shown that the cross section for a nuclear reaction is determined by the QM tunneling, and is a function of v (or equivalently of energy E), both evaluated in the center of mass frame of the reacting particles A and B. Then it must be averaged over the Maxwellian velocity distribution.

$$< \sigma v > = \; 4\pi \; [\frac{m}{2\pi kT}]^{3/2} \int_0^\infty v \; \frac{S(E)}{E} \; exp[-\frac{mv^2}{2kT}] \; exp[-\frac{-2\pi Z_A Z_B e^2}{\hbar v}] \; v^2 dv$$

Let us write $\sigma \; \propto \; exp(-b/\sqrt{E})$, where

$$b \; = \; \frac{\sqrt{2}\pi Z_A Z_B e^2 \sqrt{m_{AB}}}{\hbar} \; = \; 0.99 \; Z_A Z_B \sqrt{m_{AB}} \; (MeV)^{1/2}.$$

1a. (10 pts) To make it possible to do the integral, we decide to replace this complex integral with the integral of a Gaussian. The Gaussian is centered at E_0, with a width Δ and an amplitude C, $g(E) = C \; exp[(E - E_0)^2/(\Delta/2)^2]$. Find the appropriate parameters of this Gaussian, C, E_0 and Δ, and do the integral.

1b. (5 pts) Then define τ and derive the expression

$$< \sigma v > \propto \; \frac{1}{Z_A Z_B m} \; S_0 \; \tau^2 \; e^{-\tau}$$

2a (5 pts) We often desire to have the nuclear reaction energy generation rate expressed as a power law, $\epsilon \; = \; \epsilon_0 \rho^n T^\eta$. Consider the reaction $p+p$. Write an expression for ϵ as a function of $< \sigma v >$, ρ, etc. Use that to determine the value of n, i.e. the appropriate power of ρ. Now use the last expression given in problem 1 above to find η in terms of τ (and constants).

2b (5 pts) Energetic neutrinos from the Sun are produced by the decay of ^{8}B. Show that the rate of the reaction producing ^{8}B, $^7Be(p,\gamma)^8B$ is approximately proportional to T^{14}, when the temperature T is near $1.5{\times}10^7$ K. If one attempted to explain the factor of roughly 4 discrepancy between the observed and predicted neutrino flux in Ray Davis' experiment by postulating an error in the central temperature of the Sun, what change in the central temperature would be required ?

3 (15 pts) Assume only the main $p+p$ reactions occur in a star composed initially only of hydrogen. How long does it take for the whole chain to reach equilibrium so none of the intermediate reactions is slower than the first reaction for T and ρ that of the center of the Sun ? Plot the abundances relative to hydrogen (^{1}H) of the isotopes involved as a function of time during the time it takes to reach equilibrium under these assumptions.

4 Assume only the main pp chain and the CNO cycle main reactions are operating, and that they are at equilibrium. Use the estimate of the central temperature and central density of a star as a function of its mass obtained from the simple stellar model derived by assuming $\rho(r) = \rho_0(1 - r/R)$ you derived in the first problem of problem set 1.

4a (5 pts) Predict the total stellar energy generation rate in the center of the star via nuclear reactions as a function of stellar mass from 0.1 to 100 Solar masses.

4b (5 pts) b) Predict the fraction of energy coming from the pp chain versus that from the CNO cycle in the center of the star as a function of stellar mass.

5 (10 pts) Calculate the energy generation rate from He-burning in a pure He gas with $\rho = 10^5$ gm/cm^3 and $T = 10^8$ K. By how much would this change if the excitation energy of the 0^+ state of the ^{12}C nucleus were 7.664 MeV instead of 7.654 MeV ? (Hint: recall that this involves a resonant reaction, and see Fig. 6.11 of HKT.)

6 (15 pts) Problem 6.8 of Hansen, Kawaler and Trimble (the He flash).