Ay123

Fall 2007

STELLAR STRUCTURE AND EVOLUTION

Problem Set 5

Due Wednesday, November 14, 2006

1. (5 pts) At 500 km above the visible surface of the Sun, the temperature of the atmosphere reaches a minimum of about 4170 K. At this height, $\mu \sim 1.26$ atomic mass units and $\gamma = 5/3$. Find the scale height H and show that ω_c, which is given approximately by $c_s/2H$, corresponds to a period of oscillation of about 3 minutes.

2. Consider the restoring force from bouyancy for an element displaced upward in a star. Show that

$$F \;=\; -N^2 \Delta x = -\frac{g}{\rho}\,[\frac{\Delta\rho}{\Delta x} - \frac{\delta\rho}{\Delta x}]\;dx,$$

where $\Delta\rho$ is the difference in atmospheric density from the initial to final (upward) position while $\delta\rho$ is the same for gas inside the displaced gas, and$N$ is the bouyancy or Brunt–Vaisala frequency.

2a. (5 pts) Show that for adiabatic motions

$$N^2 \;=\; -g\;[\frac{\gamma-1}{\gamma}\frac{1}{P}\frac{dP}{dx} - \frac{1}{T}\frac{dT}{dx}]$$

2b. (5 pts) Show that N becomes 0 when the temperature gradient becomes steep enough to satisfy the condition for convection. This confirms that gravity waves due to bouyancy oscillations cannot propagate in a convective region of a star.

3. (5 pts) Estimate the Brunt–Vaisala frequency and corresponding period for the atmosphere of the Earth and of Mars. Indicate what assumptions you are making. Estimate the acoustic wave cutoff frequency ω_c, and corresponding period, for the Earth as well. What happens to a wave in the frequency interval between ω_c and N ?

4. (15 pts) Assume the Sun is fully ionized and fully convective all the way to its surface.

4a. Show that the sound speed close to the surface is given by $c^2 = (\gamma - 1)gz$, where $z = R - r << R$ is the distance from the surface.

4b. Assume a fixed horizontal wavelength k_y. (More accurately, it is l and m, the indices of the spherical harmonic, that are fixed, but if $z << R$ you can approximate $k_y \sim l/R \sim constant$.) For a given frequency ω, an acoustic wave can propagate a finite distance into the star. Find that distance z_{max}.

4c. To have a standing wave, there must be an integer number of half wavelengths in the radial direction trapped between the surface and the maximum penetration depth. This implies

$$\int_0^{z_{max}} k_r dr = n\pi$$

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Find from this a relation between n, l and ω.

4d. Compare your result with Fig. 12 of the review on helioseimology by Christensen-Dalsgaard (Revew of Modern Physics, Oct 2002, also available as Astro-ph/0207403). Check that the order of magnitude of your answer is right, and that the scaling with l and with n are correct. Print out the figure, and mark the value of n for a few of the curves shown there.

5. (15 pts) Problem 8–8 from HKT (ocean waves on the Earth)