Ay 101 - Fall 2023

Hillenbrand

Problem Set 1

due Friday, 6 October, 2023

This week we are recalling some basics from Ay 20 or your independent study.

1. **Blackbody Gymnastics**

   a. By differentiating the Planck function $B_\nu(T)$ with respect to ν, show that the peak occurs at, $h\nu_{max} = 2.82\ kT$. This ends up requiring solution of an equation of the form $x = 3(1 - e^{-x})$ which you can get past with trial and error using a calculator, or other means.

   b. Derive the Rayleigh-Jeans and Wien limiting forms of the Planck function. You should commit these frequency (or wavelength, if you prefer) scalings to your long-term astrophysics memory.

2. **Ionizing Flux.** (Leblanc #1.6) Assuming a flux distribution equal to that of a blackbody, calculate the percentage of the flux capable of ionizing hydrogen from the n=2 level, for stars with T_{eff}=5000, 10000, and 20000 K (corresponding to a late G star, early A star, and mid-B star). Note that this problem requires numerical evaluation.

3. **Solar Flux.**

   a. If $f(d_\odot)$ is the solar flux just above the earth's atmosphere and Ω is the solid angle of the Sun on the sky, show that the solar flux at the solar photosphere is $\pi f(d_\odot)/\Omega$.

   b. For $f(d_\odot) = 1.4 \times 10^6$ erg/sec/cm^2, which is a good number to memorize and use as a benchmark as you learn about radiation in astrophysics, what is the effective temperature of the Sun?

   c. Derive a general expression for the surface or "effective" temperature of the Sun in terms of: $f_\lambda(d_\odot)$, the flux per unit wavelength evaluated at one wavelength and measured at Earth; Ω; and fundamental constants.

4. **Maxwell-Boltzmann.** The three-dimensional form of the Maxwell-Boltzmann velocity distribution for particles of mass m at a kinetic temperature T is

$$N(v) = (2/\pi)^{1/2}(m/kT)^{3/2}v^2e^{-(mv^2/2kT)}.$$

   a. Show that the implied mean square velocity is $< v^2 >= 3kT/m$.

b. For main sequence stars, what is a reasonable value of m? Make a plot overlaying the velocity distributions for at least four different choices of the temperature T that roughly span the relevant range for main sequence stars. In this and all future plotting exercises, please make sure you legibly label your axes and indicate their units, where in this case I suggest km/s for the abscissa.. Also remember to use logs where they make sense.

5. **Term Project.** Start the Ay101 Term Project, beginning with the "Weeks 1-2" portion.

[**for all assignments, please write near your name how many hours you spent on the set.**]