Ay 101 - Fall 2023

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Problem Set 2

due Friday, 13 October, 2022

This week we are considering stars as gases and as gravitationally bound objects.

Note that we typically reference bulk stellar properties to the Sun, in units of e.g $M_\odot$, $R_\odot$, $L_\odot$, and that the Sun is the most well-studied star. There is a nice fact sheet on the various properties of the Sun here `https://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html` (albeit not enough cgs units......).

1. **Practice with Composition.**

   a. In a sentence or two, convey your understanding of the concepts of mass fraction (X,Y,Z) and mean molecular weight (μ).

   b. We will be using mean molecular weight when we talk about equations of state and ideal gas. Calculate the value of μ appropriate for: (1) neutral gas at solar interior composition (X=0.71, Y=0.27); (2) ionized gas at solar interior composition; (3) neutral gas at solar photosphere composition (X=0.9097, Y=0.0889); (4) pure electron gas.

   c. We will be using X,Y,Z when we talk about nuclear reaction rates. What are X and Y for a first-generation (early universe) stellar core with 92% hydrogen and 8% helium by number.

2. **Stars as Ideal Gases.**

   a. Verify from $M_\odot$ and $R_\odot$ that the average density of the Sun is slightly larger than water, and compare to the central density (calculated from stellar structure models). Soon we will cast things in terms of ρ/ρ_c when talking about stellar structure.

   b. Verify that, even at such high ρ_c, we can still consider the center of the Sun as an ideal gas. To do so, compare the Coulomb energy to the thermal energy and produce an inequality between density and temperature. Justify any assumptions and explain any adopted numbers.

   c. If all main sequence stars have roughly the same central temperature, which is actually a reasonable assumption, use a scaling argument to determine the stellar mass at which the ideal gas assumption breaks down. To do so, you first need to derive the proper scaling of $T_c \propto \mu M/R$ from either the equation of hydrostatic equilibrium or the virial theorem. Don't integrate anything, just make some simple scaling and other substitutions, stating any assumptions. Express your final answer in terms of $M/M_\odot$ and $R/R_\odot$.

d. Find a minimum value for the average temperature of a star in terms of constants, μ, M and R, starting from $< T >= \frac{1}{M} \int_0^{M_r} T dM$, rearranging some things, using the virial theorem, and stating any assumptions. Express your final answer in terms of $M/M_\odot$ and $R/R_\odot$. Why is the resulting temperature a minimum, rather than an equality?

e. Find the numerical value for the ratio of radiation pressure to gas pressure at two places, first near the center of the Sun and second in an "average place" assuming the average density $< \rho >$ and average (mass-weighted) temperature $< T >$. Also find the value for the gas pressure as a fraction of the total pressure. Soon we will refer to the latter as β and use it in our consideration of stellar structure.

f. Set your inequality in [d.] to an equality and plot the run of T with ρ for 1 $M_\odot$ stars. Choose ranges for the axes that are relevant to stars. Include lines for different μ considering each of the regimes you calculated in problem #1.

Although somewhat ad hoc and designed mainly to get you to appreciate the numbers, the results you obtained above justify our adoption of the ideal gas assumption, that we will use in the general case for pre-main sequence and main sequence stars. Post-main sequence stellar core regions, and compact object stellar remnants will have different considerations though, namely the degenerate equation of state forms that we discussed in class.

3. **Term Project.** Continue the Ay101 Term Project, finishing up on the "Weeks 1-2" portion and moving on to the "Week 3" portion. This is due at the end of the term, but please try to make progress each week.

[for all assignments, please write near your name how many hours you spent on the set.]