Ay 101 - Fall 2023

Hillenbrand

Problem Set 3

due Friday, 20 October

This week we are thinking about stars as gravitationally bound objects, the timescales that are relevant at different stages of stellar evolution, and energy sources. Although applied here in the context of star formation, these same principles will apply when we talk about stellar evolution. You should look at the notes and slides regarding timescales, which we did not cover in class.

1. **Star Formation.**

   a. Using some of the numbers given in class, calculate the free-fall timescale for a typical star forming cloud core. What stops the free fall?

   b. The gravitational energy released during the collapse is absorbed by the dissociation of H_2 molecules (4.5 eV) and then the ionization of neutral H (13.6 eV). Set up an equation for this situation involving M, R, and m_H. Now think about the thermal kinetic energy and use the virial theorem to derive the average internal temperature of the "protostar".

   c. Using some of the numbers given in class, calculate the thermal time scale, or Kelvin-Helmholtz (τ_{KH}) time. This provides a rough estimate of the duration of the pre-main sequence phase. What stops the contraction?

   d. Now assuming that R scales as $M^{1/2}$ and L scales as M^3, determine the stellar mass above which the pre-main sequence phase (from c) is shorter than the free-fall timescale (from a). How does this result help explain why it is much harder to observe and study massive stars ($> 10M_\odot$) in the process of their formation?

   e. During most of the pre-main sequence phase, the ionized gas obeys classical physics such that the temperature increases as the density increases. However, near the end, the gas becomes degenerate such that quantum mechanics becomes important, around a density of $\rho \approx \frac{\mu m_H}{h^3}(m_e kT)^{3/2}$. Thinking again about gravitational energy, kinetic energy, and the condition for virial equilibrium, show that the temperature at which the density condition above is satisfied is $T \approx [\frac{G^2(\mu m_H)^{8/3} m_e}{kh^2}]M^{4/3}$.

   f. Notice that the temperature equation in (b) is independent of mass while the temperature equation in (e) depends on mass. Any thoughts on this?

2. **Easy Interior Structure**

   a. For a star with a density profile $\rho = \rho_o(r/R)^{-2}$, find the density as a function of radius in terms of M and R. (HINT: you need solve for ρ_o).

   b. Using your answer to part (a), compute the mass m and the mean density $< \rho >$ interior to radius r.

3. **Gamow Energy.**

   a. We stated in class that the tunneling probability through the coulomb potential barrier is

$$P(tunneling) \approx |e^{-\int_{r_{crit}}^{r_{nuc}} \chi(r)dr}|^2$$

   where r_{crit} and r_{nuc} are the critical radius and nuclear radius, respectively, and χ was defined in discussing the Schrodinger equation. Show that the solution to the integral above can by found using the substitution $r = r_c cos^2\theta$ which leads to

$$P(tunneling) = e^{-(E_{Gamow}/E)^{1/2}}$$

   where $E_{Gamow} = (\pi\alpha Z_1 Z_2)^2 2m_r c^2$ with the same meanings used in class.

4. **Term Project.** Continue the Ay101 Term Project, working on the "Weeks 4-5" portion. This is due at the end of the term, but please try to make progress each week.

[for all assignments, please write near your name how many hours you spent on the set.]