**Ay 101 - Fall 2023**

**Hillenbrand**

*Problem Set 4*

*due Friday, 27 October*

This week we are thinking about energy transport in stars. How does all that $\epsilon_{nuc}$ make its way from the core to the surface?

1. **Radiative Zones.** Consider a star with radiative outer layers and for which opacity is mostly electron scattering (i.e. $\kappa = 0.2(1+X)$). Note that "radiative" means that the energy is transported by radiative diffusion, as opposed to conduction or convection; it does not necessarily mean that the *pressure* is completely due to radiation. Assume that $T = 0$ and $P = 0$ at $r = R$ and $M_r$=M in the outer layers. Show that close to $r = R$,

$$T(r) = C \times \frac{3\kappa\mu m_p}{16\pi ack} L_r(\frac{1}{r} - \frac{1}{R})$$

where $C$ is an integration constant that you should solve for.

2. **Motion of a Fluid Element.** Consider the buoyancy frequency $\omega$ where $\omega^2 = \frac{g}{\rho}(\frac{\Delta\rho}{\Delta r} - \frac{\delta\rho}{\Delta r})$ (also called the Brunt-Vaisala frequency). Define $\Delta\rho = \rho(r+\Delta r) - \rho(r)$ as the difference in density between two radial positions, i.e. $\Delta\rho$ describes the medium, and $\delta\rho$ as the increase in density of a pocket of gas that occurs when it is displaced via convective motions. Assume that the displaced pocket of gas rapidly adjusts its pressure to match the surroundings and expands adiabatically.

   a. Show that

$$\omega^2 = -g[\frac{\gamma - 1}{\gamma}\frac{1}{P}\frac{dP}{dx} - \frac{1}{T}\frac{dT}{dx}]$$

   b. Show that the buoyancy frequency $\omega$ becomes zero when the fall-off in temperature becomes steep enough to satisfy the condition for convection. This confirms that buoyancy forces in a convective region of an atmosphere do not lead to oscillations.

   c. Show that in an isothermal atmosphere

$$\omega^2 = \frac{\gamma - 1}{\gamma}\frac{g}{H_P}$$

   where $H_P$ is the pressure scale height.

3. **Adiabatic Temperature Gradient.** Show that for an ideal classical gas with $P = \rho kT/\mu m_H$ in which $\mu$ is constant throughout, and equation of state $P = P_o \rho^\gamma$ where $\gamma$ is the ratio of heat capacities at constant pressure and at constant volume, that

$$\frac{dT}{dr}|_{ad} = -g/c_P.$$

Note that taking derivatives with respect to r is a good way to start since you will then find a way to introduce $g$ into the equations.

4. **Convective or Radiative?** Near the center of a star the numerical values of several quantities are given as follows:

$$r = 0.1\ R_\odot$$

$$m(r) = 0.028\ M_\odot$$

$$L(r) = 24.2\ L_\odot$$

$$T(r) = 2.2 \times 10^7\ K$$

$$\rho(r) = 31\ g/cm^3$$

$$\kappa = 0.40\ cm^2/g.$$

   a. Neglecting radiation pressure and assuming the average gas particle mass is 0.7 amu, determine whether the energy transport is convective or radiative.

   b. Justify the assumption that radiation pressure can be neglected.

5. **Term Project.** Continue the Ay101 Term Project, working on the "Weeks 4-5-6" portion. This is due at the end of the term, but please try to make progress each week.

**[for all assignments, please write near your name how many hours you spent on the set.]**