Ay 101 - Fall 2023

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Problem Set 7

due Friday, 1 December, 2023

This set concerns our various topics in stellar atmospheres.

1. **The Shapes of Stellar Spectra.**

   Read the preprint article at `https://arxiv.org/pdf/2303.14340.pdf`.

   a. We spent most of this class discussing stellar interiors, and developing an understanding of the run of quantities like T and ρ as a function of radius for $r < R$. Consider now the Figure on page 4 (first part of Figure 1), and place the numerical values along both axes in context.

   b. Regarding the Figure on page 5 (second part of Figure 1), describe the contributions to the orange line at different wavelengths. In other words, for a typical solar-type star, which processes dominate the continuum opacity in the different wavelength ranges? Describe each of these processes in a few sentences. What is going on with the blue line??

   c. Figure 2 is a bit of a mess, but find and follow the line for neutral H across wavelengths. Describe in detail what the sharp jumps are due to. Why are the curves for the metals so much more complicated looking?

   d. Figure 3 is like Figure 1, but also includes a hotter and a cooler star. In the top panel, there are points marking the effective temperature of each star which correspond to a different density (and radius) in each case. Why are the values different for the three different objects? How is the effective temperature defined?

   e. Regarding the bottom panel of Figure 3, compared to the solar-type G star, what is different about the dominant opacity sources for the cooler M star and the hotter A star?

   f. Why does the LTE assumption simplify calculations of opacity? Where does it break down? What are some other complexities that can arise in modeling spectra?

   g. Considering Figures 4, 5, and 6 which compare models for the emergent flux to high-quality observations from space (so no contributions from Earth's atmosphere), why the mis-matches?

   h. Summarize in a few sentences how (1) surface gravity and (2) heavy element abundances affect spectral features for a solar-like star.

2. **A Theoretical Spectrum.** Now it's your turn. A reasonable representation of the continuum spectrum from a stellar atmosphere can be obtained with the understanding you have in hand. Consider a hydrogen+helium atmosphere of $T_{eff} = 5600\ K$. The lack of metals (realistic for the first generation of stars formed in the universe, but admittedly not today) means that the number of free electrons is substantially reduced compared to solar-metallicity stars, thus decreasing the contributions of H^- and free-free to the opacity. Consider only hydrogen bound-free opacity for this problem.

   Your goal is to compute the surface flux $F_\lambda(\tau = 0)$ as a function of λ over the optical portion of the electromagnetic spectrum, say $3000 - 10000\mathring{A}$, and to appreciate the prominent "Balmer jump" and "Paschen jump" in optical spectra.

   a. Assume that only the first four levels of H are populated and hence available for ionization. Calculate the relative numbers of atoms in each of the 4 levels: $N_2/N_1, N_3/N_1, N_4/N_1$ using the Boltzmann formula.

   b. Assume that $\kappa_\lambda = \kappa_\lambda(T = 5600\ K)$ independent of depth. Evaluate the opacity κ_λ *at each wavelength*, relative to that at $6000\mathring{A}$. You should write down the equivalences involving the mass opacity, the cross section, and the linear absorption coefficient to make sure you are working in the correct units. The total absorption coefficient at any given wavelength is the sum of the absorption coefficients (α_n or σ_n), times the level populations (N_n or $n_n = N_n/N_{total}$), including all energy levels that can be ionized by a photon at that wavelength.

   c. Find $\tau_{6000A} = \tau_\lambda \times \kappa_{6000A}/\kappa_\lambda$ *at each wavelength* corresponding to the depth at which $\tau_\lambda = 2/3$. Plot versus λ.

   d. Assume a grey atmosphere-like $T(\tau)$ relation. Find the corresponding temperature $T(\tau = 2/3)$. Plot versus λ.

   e. Finally, assume a linear source function and the Eddington-Barbier solution to find and plot $F_\lambda(\tau = 0)$ versus λ.

3. **Linear Source Function and Eddington-Barbier Relation**

   a. For a source function increasing linearly with optical depth, *e.g.,*

   $$S_\nu(\tau_\nu) = a_\nu + b_\nu \tau_\nu$$

   derive an expression for the second moment of the radiation field, K_ν, at the surface where $\tau_\nu = 0$.

   b. What does your result in part [a.] imply about the surface radiation pressure from photons of frequency ν? In particular, derive a relationship between this radiation pressure and the source function S_ν at some characteristic optical depth.

4. **Generalized Spectral Line Formation** Let's consider a homogeneous medium with a source function (both line and continuum) equal to a constant S_ν. The point of this problem is to qualitatively determine whether a line at ν_0 will be emission or absorption, and whether it is saturated or not

   (a) Begin by considering the general solution to the equation of radiative transfer

   $$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t_\nu)e^{-(\tau_\nu - t_\nu)}dt_\nu$$

   Assuming S does not vary with location, show that

   $$I_\nu(L) = I_\nu(0)e^{-\tau_\nu(L)} + S_\nu(1 - e^{-\tau_\nu(L)})$$

   Work through the limiting cases for large and small optical depths.

   (b) For the following cases, sketch the output specific intensity I_ν as a function of ν, in the vicinity of a frequency ν_0 where there are possible radiative transitions that could form emission or absorption lines. On each plot, also sketch the backlighting intensity($I_\nu(0)$) and source function, where they are present.

   i. The object is optically thick
   ii. The object is optically thin and not back-lit ($I_\nu(0) = 0$, $\tau_\nu < 1$)
   iii. The object is optically thin and backlit with $I_\nu(0) < S_\nu$
   iv. The object is optically thin and $I_\nu(0) > S_\nu$
   v. The object is optically thin and backlit with $I_\nu(0) < S_\nu$, but optically thick at a line center (that is, at ν_0)
   vi. The object is optically thin and backlit with $I_\nu(0) > S_\nu$, but optically thick at a line center (that is, at ν_0)

5. **Term Project.** You should be bringing this to a close soon, as it is due at the beginning of exam week.

[for all assignments, please write near your name how many hours you spent on the set.]