There is one analytic problem, and then it is time for a little programming.

1. Homology on the lower main sequence.
   
   In class we discussed L-T, M-T, and M-L scaling relationships derived from homology. We also showed an R-M scaling consistent with that measured for binary stars a bit above the mass of the Sun, \( R \propto M^{0.4} \). Using similar lines of argument, verify that the homology relations on the lower main sequence, where R-M measurements are increasingly scarce as mass decreases, are as follows.

   I am asking you to use slightly more complicated expressions than we have written in class thus far for the energy generation rate as a function of density and temperature, and the opacity as a function of density and temperature. Including the effects of composition now, try 
   \[
   \kappa = \kappa_o (1 + X) Z \rho T^{-3.5} \quad \text{and} \quad \epsilon = \epsilon_o X^2 \rho T^4 
   \]
   for Kramers form opacity and proton-proton chain fusion.

   Briefly discuss each relation below in a few words or sentence.

   a. \( R \propto Z^{0.15} X^{0.68} \times M^{1/13} \)
   b. \( L \propto Z^{-1.1} X^{-5.0} \times M^{5.46} \)
   c. \( T_{\text{eff}} \propto Z^{-0.35} X^{-1.6} \times M^{1.33} \)
   d. \( L \propto Z^{0.35} X^{1.55} \times T_{\text{eff}}^{4.12} \)

2. Gases.
   
   This is a straightforward programming / plotting exercise. Consider a star composed of classical gas with number density \( n \) in equilibrium with the radiation field at temperature \( T \). Plot the run of \( T \) with \( \rho = nm \) where \( m = \mu \times m_H = \mu / N_A \). Choose ranges in the axes that are relevant to stars. Include a clearly labelled line for each of:

   • pure electron gas
   • neutral hydrogen gas
   • ionized hydrogen gas
   • an ionized gas mixture with mass fractions \( X = 0 \) and \( Y = 0.98 \)
   • an ionized gas mixture with mass fractions \( X = 0.70 \) and \( Y = 0.28 \)

   Now, for each of the following two cases, evaluate density \( \rho \) and \( \beta \), the ratio of gas pressure to total pressure.

   • \( \log T = 7.55, \log P = 16.85 \) and \( \log T = 6.91, \log P = 16.87 \) (cgs units)
3. **Opacity Tables, Oh My.**

Develop a program for calculating the opacity in cm$^2$ g$^{-1}$ of a gas with a given density and temperature, by interpolation within a table. You should try standard solar composition first, and then also an alternate composition/mixture of your choosing. Acquire one or more opacity tables as instructed below. The sources generally give the log of the opacity, and you should interpolate in log $T$, log $\rho$ space.

What do you obtain for the opacity for log $T = 6.3$, log $\rho = 0.3$ (cgs units)? And for log $T = 5.0$, log $\rho = -4.0$? What is the bulk effect of some differing composition?

There are at least three sources for the opacity data. The first is OPAL:

http://opalopacity.llnl.gov/

Use the “Type 1” tables, which have “standard” abundance ratios for the various metals, though you should be aware that a variety of pre-computed tables with different mixtures are available, and you can also have OPAL compute opacities for you for any desired mixture.

The second source is the Opacity Project (OP):

http://cdsweb.u-strasbg.fr/topbase/TheOP.html

OP allows you to compute opacities, for any mixture, at the OPServer:

http://opacities.osc.edu/rmos.shtml

Tables are in the OPAL format, or they can be customized.

Finally, note that the OPAL/OP tables go down to only perhaps log $T = 3.5$ (maybe even a little hotter). If you are interested in exploring cooler temperatures a third source of opacities is:

http://webs.wichita.edu/physics/opacity/

These tabulations are also in the OPAL format.

[for all assignments, please write near your name how many hours you spent on the set.]