

## Ay 20 – Robinson Rooftop Observatory Exercise

This is supposed to be a fun and instructive exercise that will give you some telescope training and some insight into real science that can be done with a small-aperture telescope.

### 1. Introduction

For most of recorded history, stars were thought to be constant in brightness and eternally unchanging. As we will soon learn in class, while stars exist in a relatively steady state for >90% of their lifetimes, they do in fact undergo evolution in structure and appearance before and after this steady-state or “main sequence” phase of hydrogen-burning. Intrinsic photometric variability, i.e. observable variations in the light emitted by a star, is a characteristic associated with certain stages of stellar evolution. Famous variables include Mira and Eta Aquila. These are long period variables, the most common kind of variable star. The stars have left the main sequence and become unstable, pulsating in size and temperature. Recalling  $L = 4\pi R^2 \sigma T^4$ , this implies a change in luminosity. Such pulsating variables can change by  $\sim 4$ -6 magnitudes in the visual, or a factor of 40-250 in brightness ( $m_1 - m_2 = -2.5 \log(F_1/F_2)$ ), over the course of hundreds of days.

Photometric variability, or changes in apparent brightness, can also be caused by eclipse events in certain classes of binary star systems. The very first well-established variable star was Algol. This star varies by over 1 magnitude every 2.8 days. John Goodricke, circa 1782, correctly deduced that the regular variation of Algol was due to the star being in a binary system, and that the companion occulted some of the light from Algol once per orbit. Such variables are called eclipsing binaries. These can be categorized into three physical types: contact binaries, semi-detached binaries, and detached (noninteracting) binaries. Usually contact binaries have the shortest periods since, being more or less “in contact” as the name implies, they have the smallest orbital separations.

Indeed, the majority of stars are *not* single stars like the Sun; rather,  $\sim 2/3$  of all stars live in multiple systems (binaries, triples, quadruples, and several even higher order multiples are known). Binary stars, that is, two stars in a gravitationally bound orbit, play a central role in stellar astrophysics since by studying the dynamics of their mutual orbit <sup>1</sup> about the center of mass, we can accurately determine the masses and radii of individual stars independent of any other stellar characteristic or model. Eclipsing binaries, like Algol, are the most useful subset of the binaries for mass determination, since one can assume an approximate value for the orbital inclination  $i$  (which otherwise renders mass determination for binary systems uncertain by  $\sin i$ ). All we need to know comes from Newton and Kepler. All we need to do the work is obtain some

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<sup>1</sup>An orbit is fully specified by the following parameters: orbital period, eccentricity, semi-major axis, inclination of orbital plane relative to the plane of the sky, longitude of periastron, time of periastron passage, and longitude of the ascending node.

good quality data. Indeed, observed light curves enable us to model the luminosities, inclination angle, and radii of the component stars. Observed radial velocity curves lead to orbital radii and mass ratios. The combination of lightcurve and velocity curve data is even more powerful.

*Detached, eclipsing, double-line spectroscopic binaries* are therefore “the prize pig.” We like detached systems since they allow us to determine the fundamental parameters of normal stars (as opposed to contact systems which are potentially undergoing some sort of mass transfer process). Eclipsing binaries give us light curve information leading to stellar radii. Double-line spectroscopic binaries are systems which are close enough together and close enough in mass that we see spectral lines from both components of the binary. We can observe radial velocity variations which lead to stellar masses. When all of this occurs in the same system, we can paint a rich picture of the orbital dynamics. Note that our Sun is the *only* star for which all of the mass, radius, luminosity, and age can be determined directly and reliably. Only for a few other stars is *any* fundamental mass and radius information available. We use these kinds of systems to calibrate relationships between fundamental properties and observables, which are then applied to most stars (e.g. the mass-luminosity relationships along the main sequence that we talked about in class).

Many eclipsing binaries are bright ( $V < 10$  mag), <sup>2</sup> have short orbital periods ( $< 1$  day) and have deep eclipse minima ( $> 0.1$ - $0.2$  mag at  $V$ ), making them easy targets for even modestly equipped observatories such as our 14” rooftop telescope and CCD equipment. In general, the scientific goals of eclipse timing observations are to determine the period with high accuracy so that eclipses can be accurately predicted, and to detect period changes which might be caused by physical changes in the system, e.g. mass transfer or presence of a third body in the system. For our experiment, however, we will be satisfied with reproducing the results of other researchers for a small number of eclipsing systems.

For this project we will pick out several eclipsing double-line spectroscopic systems and attempt to combine lightcurve data taken on the rooftop with published information on relative radial velocities, and derive *fundamental* stellar parameters.

## 2. Goals of this Exercise

The main goal of this Robinson rooftop observing exercise is to introduce you to the basics of

- obtaining astronomical data;
- performing simple “reduction” of images;
- deriving photometry;

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<sup>2</sup>Recall that Vega is 0<sup>th</sup> magnitude, the Trapezium stars in Orion are 5<sup>th</sup> magnitude, and the faintest objects seen with Keck are about 28<sup>th</sup> magnitude.

- taking some top-level analysis steps;
- combining what is learned from the new data with existing information from the literature to draw your own conclusions.

We will be observing several eclipsing systems through the Robinson Rooftop Observatory 14” telescope. The rooftop is actually a terrible site for astronomy for several reasons, including: *substantial* light pollution (both from lights hanging from the building itself and from the general glow of Pasadena/LA), total occultation of certain parts of the sky by nearby buildings (especially Millikan), low altitude (more atmospheric water vapor than at higher sites), and horrendous seeing (especially to the south towards physical plant where there are heat and air currents being generated). So the observing conditions will not be “photometric” by a long shot. But we can still do some observing that is at least instructive and (hopefully) fun.

There are several filters available, approximating the “B” (3900-5050Å), “V” (4950-5800Å), and “R” (6100-6700Å) filters we talked about in class. We will process the raw data and work with the resulting images to extract photometry. With the photometry we can make lightcurves, i.e. plots of magnitude vs time. Subsequent analysis will lead us to estimates of stellar parameters.

### 3. Preparing for Observing

#### 3.1. Object Selection

A list of eclipsing systems that are relatively short period, bright enough to observe, and “up” during the fall has been chosen (see table below). Every observing group should obtain data on as many of these stars as deemed worthy, based on the most return for telescope time spent. You should think about availability of the target based on its coordinates and the ease with which you can observe an eclipse. An indication of “rv” in the last column means that radial velocity data are also available for the source. In reality only 2-3 stars per observing session will be feasible, so choose wisely. For each star, plan to take 4-5 sets of data per night. As described below, a set should consist of 5-7 exposures, with the intention to average these measurements after the photometry is extracted. The exact dates and times of upcoming eclipses for a few of the stars on this list can be found at: <http://capella.physics.gatech.edu/eclipses>. Especially if data from all the groups can be combined, we should be able to make some nice lightcurves. Make sure you do the following *before* coming to the telescope.

#### 3.2. Other Information to Assemble

Using resources in the library and on the Web, look up relevant data for all objects you are considering observing. In particular, search for equatorial coordinates, in J2000.0 if possible, which

will help you decide which are the optimal targets on your observing data (in the end, I decided to include these for you in the table). You might also look for magnitudes, colors, spectral types, proper motion data, and any other information you think will be relevant for our analysis.

Name	HD or HIP	R.A. and Dec. (J2000.)	V mag range	period	eclipse duration	comment
V471 Tau	HIP 17962	03 50 24.9 +17 14 47	9.5-9.7	0.52 day	0.8 hours	total eclipse; rv
MR Del	HD 195434	20 31 13.4 +05 13 08	8.8-9.1	0.52 day	1.9 hours	
SV Cam	HD 44982	06 41 18.7 +82 16 02	8.4-??	0.59 day	?	total eclipse; rv; not up?
RT And	HIP 114484	23 11 10.1 +53 01 33	9.0-9.8	0.63 day	3 hours	total eclipse; rv
HS Aqr	HD 197010	20 40 54.0 -00 35 50	8.9-9.2	0.71 day	3.4 hours	rv
UV Psc	HD 7700	01 16 55.1 +06 48 42	9.0-9.9	0.86 day	4 hours	partial eclipse
X Tri	HD 12211	02 00 33.7 +27 53 19	8.9-11.9	0.97 day	hours	period changes?
IQ Per	HD 24909	03 59 44.6 +48 09 04	7.7-8.2	1.74 days	4.2 hours	rv
PV Cas	HD 240208	23 10 02.6 +59 12 06	9.7	1.75 days	hours	
V442 Cyg	HD 334426	20 27 52.3 +30 47 28	9.7	2.39 days	hours	rv
EE Peg	HD 206155	21 40 01.9 +09 11 05	7.0-7.6	2.63 days	6 hours	triple system; rv
CW Cep	HD 218066	23 04 02.2 +63 23 49	7.6-8.0	2.73 days	9 hours	rv
V 478 Cyg	HD 193611	20 19 38.7 +38 20 09	8.6-8.8	2.88 days	hours	
TRES-1		19 04 09.8 +36 37 57	11.8 +3%	3.03 days	3 hours	0.75 $M_{Jup}$ planet!
HD 209458	HIP 108859	22 03 10.8 +18 53 04	7.7 +1.5%	3.52 days	3.2 hours	rv; 0.7 $M_{Jup}$ planet!
$\lambda$ Tau	HD 25204	04 00 40.8 +12 29 25	3.3-3.9	3.95 days	14 hours	triple system; rv
$\beta$ Per = Algol	HD 19356	03 08 10.1 +40 57 20	2.1-3.4	2.87 days	10 hours	prototype; semi-detached binary
$\beta$ Lyr	HD 174638	18 50 04.7 +33 21 46	3.4-4.2	12.94 days	4 days	prototype; contact binary

Now, what is the calendar date of your observing session? Roughly, what range of right ascension values are "up" (think about the local sidereal time at midnight and the number of dark hours in the night)? Using this information and looking at the coordinates, decide which stars you want to observe.

Next, using the coordinates, create a finder chart for your stars using the online Digitized Palomar Sky Survey (DPOSS) at [http://archive.stsci.edu/cgi-bin/dss\\_form](http://archive.stsci.edu/cgi-bin/dss_form). Only one of these is needed per group; either work together or distribute tasks over the course of the project. The finder will be used to determine the location and orientation of the CCD field of view during your observations and should be large enough to encompass the field of view of the CCD in any orientation ( $\sim 10'$  for the f/3.3 focal reducer and  $\sim 1^\circ$  for the f/6.6 focal reducer). As a starting point, the pixel scale of the detector is 0.6 arcseconds/pixel and the size is  $512 \times 512$  pixels. You should request a .gif image instead of the default .fits (unless you want to deal with saving and printing .fits). This image will also be used to identify good comparison stars which are needed for our technique of relative or differential photometry. The comparison stars are probably slightly – but not substantially – fainter than the target star, and well-isolated in the field, that is, not

located in a crowded region. Are there suitable comparison stars in the field? If not, you might want to make the image a little bigger until you find one.

Finally, what is the Julian Date? This is defined as the number of days which have elapsed since noon (UT, that is Universal Time) on 1 January 4713 B.C. As a hint I will tell you that  $JD = 2450000.0$  occurred at noon (UT) on 9 October 1995. Remember that UT is equivalent to GMT, and PST is 8 hours behind that. So  $00^h$  UT occurs in the late afternoon here and the Julian Day starts in the middle of the night. It is customary to express time series observations in fractional Julian Days, abbreviated JD. Initially you need know only the integer day; you will fill in the decimals after you acquire the data.

#### 4. Observing Session

The f/3.3 focal reducer is recommended in order to avoid vignetting around the edges of the field. The TAs will guide you through the steps to startup and point the telescope, which involves setting the telescope to a star of known position and telling the computer where you think you are. The final startup steps are to focus the telescope and initiate the CCD. The TAs will also help you acquire the objects in the finder and place them onto the detector for digital data acquisition. Exposures of up to 2-3 minutes are possible and get us down to about 15<sup>th</sup> magnitude at V-band. Longer integration times are not advised due to the relatively poor accuracy of the tracking (constant change of the telescope's azimuth and elevation in direct compensation for the earth's rotation) and the lack of a guider (which would correct for poor tracking by keeping a "guide star" in a fixed position). You can try a long exposure if you want; the effect will be images that are trailed (i.e. smeared) in the east-west direction. Most important before moving on to the next step: get out your notebook and record what is going on! Some good things to keep track of once you start taking data frames are: file number, time of observation, filter, exposure time, and any relevant comments. Again, be sure to distribute the work amongst your group members; the person who made the finding charts should not have to keep log too.

First, perhaps even as part of the focus process, go to the double star list at <ftp://ftp.lowell.edu/pub/bas/dbls.fil> and look at several of these systems. What is the closest system you can resolve? How does this compare to the diffraction limit of the 14" telescope? Welcome to the poor "seeing" below 5000 ft elevation.

Next, go to the first object on your observing list. Take a short exposure and check the number of counts in the resulting image. It will take about 1 minute for the detector to read out and the computer to record the image on disk. Decide on an appropriate exposure time that will put plenty of counts on the image but not so many that the detector saturates. Take about 7-10 exposures of your first star. The sets of 7-10 measurements, if they are all suitably good, will be averaged later in order to beat down the background noise and hence improve the signal-to-noise ratio in your photometry. Look at the images to see if the object you picked as a comparison star

is there, or if there are other suitable comparison stars present. If not, move to your comparison star and take another 7-10 images of it. If it is in the same image as the target star, think about whether you want to observe in more than one filter or not, and if so, do it. Move on to the next object and repeat the procedure. After observing several stars on the program from Table 1, come back and re-observe each one. Do this several (4-5) times so that you sample the lightcurve of any given star over several hours if possible.

Finally, you should also take some calibration frames. Flat-field frames are obtained by holding the white foam dish several feet in front of the telescope and exposing for 5 seconds with the lights on. The point is to have out-of-focus scattered light enter the telescope in a uniform illumination. It is best to take several of these (5-10) and average them together. These flat fields are used to correct for pixel-to-pixel response differences in the CCD. Also take some bias frames, or 0 second exposures. These are used to remove the zero-light count level on the detector.

## 5. After Observing

### 5.1. Notes to Make

Make a brief narrative summary of the observations for your night, including:

- The date of the observing, and the name of the people in your group (including the TA).
- The approximate conditions (seeing and weather) during the night. Note any changes that occurred over the observing period (e.g. clouds moving in, or moon rising).
- The starting and ending time of the observations. This will be from the UT (or LST) time of the first image until the UT (or LST) time when the last image was recorded. Also record the range of airmass (given as “SecZ”, the secant of the zenith distance).
- Estimate the total elapsed time required to execute the observations, i.e. the sum of all integration times.
- Estimate the “observing efficiency” by computing the ratio of the amount of time spent actually collecting photons to the total amount of time needed to execute the observations. Telescope time is valuable, so you generally want this number to be as high as possible; at Keck, for example, this number can be as high as 90% for some programs.

### 5.2. Reduce the Data

*Preliminary Steps:* Each image contains your source plus a number of other (probably fainter) stars which we will try to use as comparison stars. Alternately, you may have taken specific

exposures of well-chosen comparison stars. So far as we know at the outset (and will learn more about them as we proceed), these stars are not variable. The comparison stars are used for relative or differential photometry. We use this technique because of the generally nonphotometric conditions, that is, changes in time and on short spatial scales of the atmospheric transparency from Pasadena. For a single image, however, the time effect is mitigated and there should not be much structure on the scale of a few arcminutes, the size of the CCD.

Other features on these images are various blemishes that will appear in all frames, and bright pixels which mark the passage of a cosmic ray through the CCD during the exposure and appear in different places from frame-to-frame. These leave behind bright, often saturated, spots that look different from stars in that they are more sharply spiked than the roughly gaussian appearance of stellar point sources.

The TAs will help you subtract the bias frame (which accounts for electrons trapped in the potential well of the detector pixels in the absence of light incident from the telescope optical path) and divide by the flat-field (which corrects for differences between the individual pixels in their response to light) right there on the computers at the telescope. If you have taken your images in sets (through the same filter), do **not** average them at this point. Any averaging should be done after the photometry is extracted. For kicks, if you have taken images through multiple filters, you might make a color image from, say B, V, and R images. Most of the stars in the field are probably approximately the same color, but there may be a few particularly blue or particularly red ones that stand out.

*Photometry.* You should use the “CCDsoft” software which is currently installed on the PC in the dome and on one of the PCs in the Robinson computer room (052). Choose an image from among your first variable star data set and display it. Identify the targetted variable and at least 2 (preferably several) fainter stars to serve as comparison stars. The comparison stars are used to remove any variations in transparency (due to airmass or clouds) between images on a given night or between nights if you have had multiple observing sessions. Since the variable and the comparison stars are observed simultaneously on each image, they are affected exactly the same way by these phenomena. Now measure the typical seeing as recorded on your images (i.e. what is the FWHM value, first in pixels and then converted to arcseconds?). Compare this value to your estimates of the closest resolvable binary made at the telescope. The next step is to extract photometry for your main object and the one or more calibration stars in the field.

The CCD produces and records as its output “counts” or “ADUs” (analog-to-digital units) which are linearly related to the photons that travelled many many light years from their source to hit our detector. There is software available for measuring the integrated intensity of a star over all of the detector pixels that it covers (due to a combination of the instrumental point-spread-function and the atmospheric seeing conditions). You will want to measure the brightness of the stars by adding up counts in circular apertures centered on each star, and using an annulus of pixels outside this aperture containing the star, to estimate the sky level around

each star, as follows.

Make sure the star is centered in the aperture and that your aperture is big enough; you will underestimate the signal if counts in the wings of the point-spread-function (PSF) are not included. Play with the image stretch to see how far the PSF discernably extends. Part of the photometry process is subtracting from the integrated intensity measurement a “sky” measurement that includes contributions from artificial sources here on the ground (light pollution), moonlight, sunlight scattered from interplanetary dust (zodiacal light), and starlight scattered from interstellar dust. If none of these “sky” terms were present, the image would have very few counts in it other than where the stars were located. Sky subtraction is best done by defining an annulus directly around the photometry aperture for each star; it can also be done by taking a measurement in a beam that is offset from each star and does not include any other stars. If the flat-fielding is very very good, you can also produce just a single sky measurement and apply it to all stars, but this is not recommended. Make sure that you record your output in magnitudes, or  $-2.5 \times \log(\text{counts})$ .

At this point you should have measurements for each variable star and its corresponding (or several) comparison star(s). You might take this opportunity to average the sets of 7-10 measurements you made; also compute the uncertainties on the magnitudes as the standard deviation of the mean. You will also have to convert these “instrumental magnitudes” into calibrated magnitudes by finding the difference between your instrumental magnitudes and the published magnitudes for the same stars, and then applying that difference to all stars. You can use the table as a rough guide. Most important for this exercise are the differential magnitudes that are measured from the established baseline; we care less about the absolute calibration.

From the averaged magnitudes (computed in sets of 7-10), you now want to calculate *differential* magnitudes for each averaged observation. You should have done 4-5 sets of observations and should perform the following for each set:

- $V_{\text{diff}} = m_{\text{variable}} - \frac{1}{N_{\text{comparison}}} \times (m_{\text{comparison } 1} + m_{\text{comparison } 2} + m_{\text{comparison } 3} + \dots)$
- $C_{\text{diff}1} = m_{\text{comparison } 1} - m_{\text{comparison } 2}$  ;  $C_{\text{diff}2} = m_{\text{comparison } 2} - m_{\text{comparison } 3}$  ; etc.

The first ( $V_{\text{diff}}$ ) is the differential magnitude of the variable star relative to the comparison stars (two, three, or more). We use this combination to attempt to reduce the effects of random errors on any one of the measurements. There are fancier, and arguably better, ways of doing this, but this is good enough for our purposes. The second number ( $C_{\text{diff}}$ ) is the difference between the magnitudes of the two comparison stars, and tests to see if the comparison stars truly do not vary. If nothing else, tracking this value lets you set quantitative limits on the degree of their variability relative to that of the target star. In cases where the variability is weak, the statistics of the check-star differences let you assess the significance of any variability you detect, or let you place limits on variability if none is apparent.

Next, extract the *time* of each data point from the image headers, and convert it into the Heliocentric Julian Date at the middle of the time series (basically this is the time of the middle

exposure, i.e. number 5 or 6 out of the series of 10). You would like several decimal places on the date; figure out the appropriate accuracy. If you are lazy, there are Julian Day calculators on the web.

The last step will be to compile a table with your measurements. Include one row for each averaged set of 10 measurements and list at least the following five columns:

Star Name	HJD-2450000	Vdiff	Err	Cdiff1	Err	.....
[ ]	[day]	[mag]	[mag]	[mag]	[mag]	
blah	1813.12345	-5.123	0.056	0.535	0.065	

This table will form the basis of the subsequent photometric analysis.

### 5.3. Analysis

Depending on how the observing goes, you may want to work with just your own group's data, or combine with that obtained by other or all groups.

The first thing to do is plot the lightcurve, that is, brightness vs time. Remember that brighter stars have smaller magnitudes and that hotter, bluer stars have smaller colors. All of your plots should have magnitude axes that go from faint to bright, and color axes (if you make any color plots) that go from blue to red. Qualitatively describe any features you see in the time series photometry.

In order to familiarize yourself with the richness of information contained in your data you might want to peruse <http://www.physics.sfasu.edu/astro/ebstar/ebstar.html>. We will be covering similar material in lecture during the week on binary stars and fundamental stellar parameters. In what follows we are going to assume circular orbits, which is not always true, but safe here to first order. This means that  $e$  (eccentricity) = 0 and orbital velocities are constant at all times. Some runnable codes (including downloads) that generate lightcurves can be found at <http://www.physics.sfasu.edu/astro/binstar.html>. These are fun to play with and give you a sense for the effects of various parameters on observed lightcurves. What is the minimum value of  $i$  (inclination) that is consistent with your lightcurve data?

The first thing to determine is the orbital period of the system. Usually this is done by fourier techniques resulting in a periodogram, or distribution of the power at different frequencies. For our purposes, by-eye period estimates from your plot of magnitude vs Julian Date are good enough. What is the period, roughly? By carefully measuring the central time of primary (deepest) minimum and comparing with previous observations of the period and a reference JD (called JD0), one can in principal, determine whether the orbital period is constant.

Now, as Kepler told us,

$$P^2 = \frac{4\pi^2(a_1 + a_2)^3}{G(M_1 + M_2)}$$

(where  $a_1$  and  $a_2$  are the semimajor axes of the two stars relative to the center of mass, and  $M_1$  and  $M_2$  are the individual stellar masses. Using the observed lightcurve, determine the radius of each star,  $r_1$  and  $r_2$ . Usually there is lots of degeneracy in apparent lightcurves with respect to different geometries (radii, inclination angle) though if the eclipse is near-total this is better than for a more grazing eclipse. You may want to refer to some of the simulators referenced above.

Now incorporate literature information provided on the web page or that you find yourself. Using a plot of radial velocity vs time (or vs phase) for your star, first estimate the radial velocity of the system's center of mass, i.e. the mean velocity of the system. This is usually called the  $\gamma$  velocity and is actually only of secondary importance. Now estimate  $v_1$  and  $v_2$ , the orbital velocity of the primary and secondary binary components around the center of mass, assuming circular orbits. This is just the amplitude of the radial velocity curve for each component (also called  $K_1$  and  $K_2$ ). Using these numbers, determine the mass ratio of binary, remembering  $M \sin^3 i = K^3 P$ . Now calculate the semi-major axes,  $a_1$  and  $a_2$ . Finally, remembering conservation of angular momentum you can, at last, calculate the masses of the two components individually,  $M_1$  and  $M_2$ , instead of just the mass ratio or the sum of the masses.

The last point of interest is to test the degree to which the system/s you have observed really are detached, that is, noninteracting. The standard parameterization for this is the Roche lobe radius, or

$$R_{Roche} = \frac{0.49a}{0.6 + q^{-2/3} \ln(1 + q^{1/3})}$$

In this equation,  $a$  is the separation and  $q = M_2/M_1$  is the mass ratio. Compare the Roche lobe radius for each system you have observed to each of the stellar radii,  $r_1$  and  $r_2$ .

## 6. Assignment

It really is best if you don't procrastinate on this project. We would like to stagger observing sessions through the term to avoid problems with weather (more likely the later it gets in the year). The TAs will arrange the observing sessions and help with the initial data reduction. The analysis can be done either in groups or individually, as you wish. You can probably start on the analysis the same night you take your data. But the full set of steps may require multiple trips to the Robinson PC until you are happy with the results of the photometric extraction. The subsequent analysis can be done using your favorite plotting and analysis software elsewhere on campus.

A 2-3 page write-up of your activities at the telescope and your analysis of the obtained data is due by 8 December 2003, the Monday following the last day of classes. Since this is not a lab

class I do not have any formal requirements on report format. A clearly written and detailed summary is all I am looking for. Be sure to include all relevant plots you have made, and to answer within the discussion the many questions I have asked throughout this document. Discuss sources of error at all relevant stages. Finally, how could we improve this experiment?