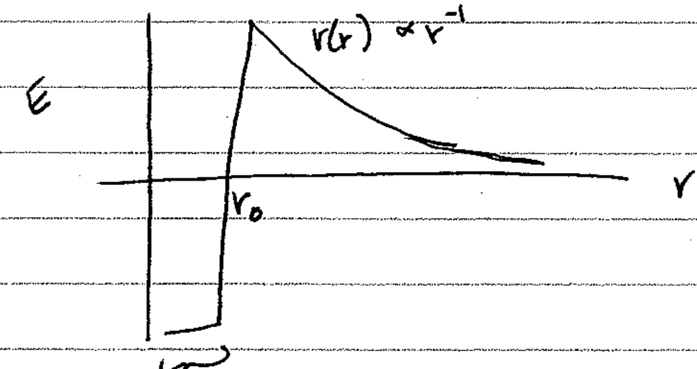
Office Hours

11/01/13

Nuclear Reactions

Nuclear reactions are the primary source of power in stars → on the MS, it is the fusion of 4 ^1H into ^4He. For fusion to occur, an atomic nucleus must overcome the Coulomb barrier of another nucleus, moving to a radius where the strong force dominates.



$r_0 \approx 1.4 \times 10^{-13}$ cm.

strong force regime.

In the classical picture, the ability of a particle to overcome the Coulomb barrier is determined by whether the thermal energy is greater than the Coulomb potential energy. The temperature required to overcome the barrier can be estimated using the following equation:

$$\frac{3}{2} k T_{classical} = \frac{Z_1 Z_2 e^2}{r}$$

This gives $T_{classical} \sim 10^{10}$ K. assuming r is the radius of a nucleus ($\sim 10^{-13}$ cm).

Since a typical internal stellar temperature is $\sim 10^7$ K, the characteristic kinetic energy (~ 1 keV) is $\sim 10^{-3}$ of the energy required to overcome the Coulomb barrier.

Question: Can nuclei in high energy tail of MB distribution overcome the barrier classically?

The fraction of nuclei with such high energies is:

$$e^{-E/kT} \approx e^{-1000} \approx 10^{-434}$$

The # of protons in the Sun is:

$$N_p \approx \frac{M_\odot}{m_H} = \frac{2 \times 10^{33} g}{1.7 \times 10^{-24} g} \approx 10^{57}$$

Thus, there is not a single nucleus in the Sun (or in all the stars in the observable universe), with the KE required to classically overcome the Coulomb barrier and undergo nuclear fusion.

The answer is QM tunneling, whereby the inherent uncertainty in the position of the particle is large enough that one nucleus can wind up in the potential well of the other and allow fusion to proceed.

The probability of successful tunneling is related to the ratio of the Coulomb barrier height to the initial Kinetic energy of the incoming nucleus, and is given by:

$$g(E) = e^{-\sqrt{\frac{E_G}{E}}}$$

Where E is the particle KE and E_G is the Gamow energy

$$E_G = (\pi \alpha Z_1 Z_2)^2 \, 2\mu c^2$$

fine structure cnst   reduced mass

The probability for a nuclear rxn, however, will depend not only on the prob. of penetrating the Coulomb barrier, but also on the nuclear X-section:

$$\sigma(E) = \frac{\overset{\text{cnst, weak fn of energy}}{S(E)}}{E}\, g(E) = \frac{S(E)}{E}\, e^{-\sqrt{\frac{E_G}{E}}}$$

$S(E)$ is derived from accelerator experiments, or calculated theoretically

strong force $\quad f_{nuc} \propto \frac{S}{r^2} \exp\left(\frac{-r}{r_0}\right) \quad S_{strong} \sim 10$

The rate of nuclear rxns is given by:

$$R_{AB} = n_A n_B \sigma_{AB} v_{AB}$$

If the average energy per rxn $\Delta mc^2 = Q_{AB}$, then the energy generation (power per unit mass) is:

$$\epsilon = \frac{Q_{AB} R_{AB}}{\rho} = \frac{Q_{AB} n_A n_B}{\rho} \langle \sigma_{AB} v_{AB} \rangle$$

Since the nuclei in a gas will have a distribution of velocities, every velocity has some prob. of occurring. So ϵ can be obtained by averaging $v_{AB} \sigma_{AB}$ over all velocities, with each velocity weighted by its probability, $P(v_{AB})$.

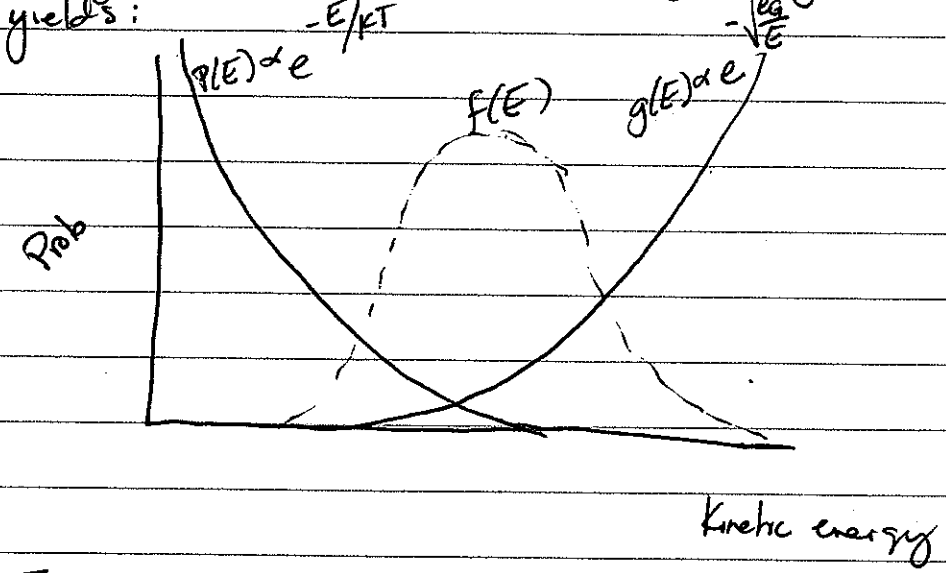
$$\langle \sigma_{AB} v_{AB} \rangle = \int_0^\infty \sigma_{AB} v_{AB} \underbrace{P(v_{AB})}_{\text{MB distribution}} dv_{AB}$$

After doing a bit of math, you can show that:

$$\langle \sigma v \rangle \propto \int_0^{\infty} \underbrace{e^{-E/kT} e^{-\sqrt{\frac{E_G}{E}}}}_{f(E)} dE.$$

This integrand is composed of two exponential functions, one from the MB dist and one from the tunneling prob.

Plotting the two functions separately along with the integrand yields:



This curve describes the range of kinetic energies over which nuclear reactions will take place.