

Solutions for Midterm 2009 October + Suggested grading

1. Total points = 20 (2 per definition). Answers pretty obvious? (*let me know if not*)
- (a) 4 min earlier
 - (b) An apparent displacement or difference of orientation of an object viewed along two different lines of sight, and measured by the angle or semi-angle of inclination between those two lines.
 - (c) Absolute magnitude measures a celestial object's intrinsic brightness. To derive absolute magnitude from the observed apparent magnitude of a celestial object its value is corrected from distance to its observer. Absolute magnitude then equals the apparent magnitude an object would have if it were at a standard luminosity distance (10 parsecs) away from the observer, in the absence of astronomical extinction.
 - (d) The effective temperature of a body such as a star or planet is the temperature of a black body that would emit the same total amount of electromagnetic radiation.
 - (e) First point (or cusp) of Aries and first point of Libra are archaic names used by navigators and astrologers for Vernal and autumnal equinox.
 - (f) The images of two different points are regarded as just resolved when the principal diffraction maximum of one image coincides with the first minimum of the other. This is an empirical diffraction limit is given by the Rayleigh criterion
 - (g) The distance modulus is a way of expressing distances that is often used in astronomy. The distance modulus $\mu = m - M$ is the difference between the apparent magnitude m and the absolute magnitude M of an astronomical object.
 - (h) Wien's displacement law states that the blackbody curve at any temperature has essentially the same shape as the curve at any other temperature, except that each wavelength is displaced, or moved over, on the graph. From this general law, it follows that there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature.
 - (i) The plate scale of a telescope connects the angular separation of an object with the linear separation of its image at the focal plane.
 - (j) Optical depth, or optical thickness is a measure of transparency, and is defined as the negative logarithm of the fraction of radiation (or light) that is not scattered or absorbed on a path.

2. Total points = 20

(a: 2 points b: 5 points c: 4 points d: 5 points e: 4 points)

- a) If the limiting parallax $\theta = 0''.005$ mas, this implies that $0''.005$ is the resolution available for the parallax measurement

$$\therefore d_{\max} = 1''/0''.005 \text{ pc} = 200 \text{ pc}$$

b) Mean distance between stars is 2 pc. \therefore Each star occupies a sphere of mean radius 1 pc, volume = $4/3\pi \times (1\text{pc})^3 = V_{\text{star}}$

From part a), volume in which we can measure parallax accurately = $4/3\pi \times (200 \text{ pc})^3 = V_{\text{sphere}}$ \therefore Number of stars whose distance can be measured = $V_{\text{sphere}}/V_{\text{star}} = (200)^3 = 8 \times 10^6$ stars with measureable parallax

- c) $\theta_{\min} = 0''.5$ and $\lambda \sim 6000\text{\AA}$.
 And for single aperture telescopes, $\theta_{\min} = 1.22\lambda/D$
 $\therefore D = 1.22\lambda/\theta_{\min(\text{radians})} = 1.22 \times 6000 \times 10^{-8} / 2.42 \times 10^{-6} \sim 30.1 \text{ cm}$

Not very impressive! That's seeing for you.

Diameters of existing large single aperture telescopes: Palomar 5 m, Keck 10m, Gemini 8m, Subaru, VLT etc

- d) For interferometers, $\theta_{\min} = \lambda/d$ where d is the largest separation of the telescopes
 i.e. interferometers can resolve objects separated by angular distance λ/d .
 Here, $\lambda = 10 \mu\text{m} = 10 \times 10^{-6} \text{ m}$ and $d = 80\text{m}$
 $\therefore \theta_{\min} = 10 \times 10^{-6} / 80 \text{ radians} = 1.25 \times 10^{-7} \text{ rad} = 0.026''$
 The maximum distance R to the star at which linear separation of star and planet is 5 AU, is then given by $\tan \theta_{\min} = 5 \text{ AU} / R$,
 $\therefore R = 5 \times 1.5 \times 10^{13} / \tan \theta_{\min} = 4 \times 10^7 \text{ AU}$.

- e) Again $\theta_{\min} = \lambda/d = 5500\text{\AA} / \text{separation of elements of interferometer(baseline)}$
 $\therefore \text{baseline} = 5500\text{\AA} / \theta_{\min}$
 $P'' = 1/D(\text{pc})$ where D is distance to object and $P \geq \theta_{\min} = \lambda/d$
 $\therefore \text{baseline } d \geq \lambda (1/D \text{ radians})$
 $\therefore \text{baseline} \sim 5500 \times 10^{-8} / 7 \times 10^{-12} = 79 \text{ m}$

Thus separation must be at least 79 m.

3. Total points = 20

(a: 2 points b: 3 points c: 5 points d: 4 points e: 6 points)

- a) $d = 1/P'' = 250 \text{ pc}$
 b) $m = 2^m.85$ and $d = 250 \text{ pc} \therefore M = m - 5\log(d/10) = -4.14$
 c) $M_{\text{bol}} = M + \text{Bolometric Correction (BC)}$. And $B-V = -0.31 \therefore$ in tables, take average BC between $B-V = -0.4$ (BC = 4.1) and $B-V = -0.2$ (BC = 1.9)
 $\therefore \text{BC} \sim 3.1$
 $\therefore M_{\text{bol}} = M + \text{Bolometric Correction (BC)} = -4.14 + 3.1 = -1.14$
 d) Compare Sun and star: $L_*/L_{\odot} = 100^{(M_{\odot} - M_{\text{bol}})/5} \therefore L_* = 201.372 L_{\odot} = 7.65 \times 10^{28} \text{ W}$
 e) A B0V star such as T Sco should have strong HeI absorption features. Balmer lines will be weakening compared to A stars but still quite strong. A) stars a "white light" stars characterized by the strongest Balmer absorption lines and fairly strong CaII lines. M dwarfs are the coolest red stars; their spectra are dominated by molecular absorption bands, especially TiO as well as metal absorption lines

4. Total points = 20

a: 5 points b: 4 points c: 4 points d: 3 points e: 4 points)

- a) For plotting rough diagram, Sun's position and points A, B, and C correctly give 5 points total
- b) $L = 4\pi R^2 \sigma T^4$ so $L_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4 \therefore R_*/R_{\odot} = (L_*/L_{\odot})^{1/2} \times (T_{\odot}/T_*)^2$
For Sun, use $T_{\text{eff}} = 6000\text{K} \therefore R_* = (54)^{1/2} \times (6000/9250)^2 = 3.1 R_{\odot} = R_A$
- c) $R_B = 3.63 R_{\odot}$ and $R_C = 71 R_{\odot}$
- d) Spectral types OBAFGKM represent a temperature scale with O hottest and M coolest. Although Stars A, B, C, have different radii and luminosities, they all have temperatures around 10^4 K, thus same spectral type, with similar spectral shapes and strong emission from Balmer lines of hydrogen.
- e) Since star D has $T_{\text{eff}} = 6000$ K, it has same spectral type as the Sun, a G2 star. $R = 10 R_{\odot}$. \therefore from relation between L, T and R above, $L \sim 100 L_{\odot}$ ($116.915 L_{\odot}$). H-R diagram position suggests luminosity class is therefore III.

5. Total points = 20

a: 2 points b: 9 points c: 9 points)

- a) $B_{\lambda}(T) = 2hc^2/\lambda^5 \times 1/(e^{hc/\lambda kT} - 1) = 2.55 \times 10^{14} \text{ ergs/cm}^2/\text{sec/cm/steradian} = 5.21 \times 10^{-17} \text{ ergs/cm}^2/\text{sec}/\text{A}^{\circ}/\text{steradian}$
- b) If Sun is a BB at 5760 K and $m_{\odot} = -26.80$ at 5500Å and star has $m = 0.0$, the ratio of fluxes, $F/F_{\odot} = 10^{(m_{\odot} - m)/5} \therefore F = 100^{-26.8/5} F_{\odot} = 100^{-26.8/5} \times \pi B_{\lambda}$.
Thus $F = 15277.9 \text{ ergs/cm}^2/\text{sec}$.
Photons of wavelength 5500 Å have energy $h\nu = hc/\lambda = 3.6 \times 10^{-12} \text{ ergs/sec}$
 \therefore Number of photons = $F/h\nu = 4.2 \times 10^{15}$.
- c) The Balmer jump is the abrupt drop in the continuous spectrum at wavelength 3647 Å. It results from the fact that any photon with wavelength below 3647 Å can be absorbed by electron in the n=2 level of hydrogen. The opacity increases abruptly. The U filter is centered very close to this wavelength so that U-B color is heavily affected for star with strong Balmer lines. This causes a significant deviation from the blackbody line in a color-color diagram.

6. Total points = 5

End of September is close to autumnal equinox, with local noon = 12h LST and midnight = 0h LST (2 points)

The Orion constellation is at RA $\sim 5^{\text{h}}$, so it is on the meridian (overhead) only at 5 am. In September its northern stars can be seen from Spain after about midnight. (2 points)

In photographs of Orion, Betelgeuse ($\delta=05^\circ$) is much further north than Rigel ($\delta= -05^\circ$) and will appear first. (1 point for knowing Betelgeuse further north; declinations need not be quoted)