AY 20: Basic Astronomy and the Galaxy

Solution Set 6

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1. C&O Problem 12.1

(a) Apparent visual magnitude of a star without extinction

$$m_V \to M_V + 5\,\mathrm{Log}[10, d] - 5 \;/.\; \{M_V \to -1.1,\ d \to 700\}$$

$$m_V \to 8.12549$$

(b) Apparent visual magnitude of star if lying behind the nebula

$$m_V \to M_V + 5\,\mathrm{Log}[10, d] - 5 + A_V \;/.\; \{M_V \to -1.1,\ d \to 720,\ A_V \to 1.1\}$$

$$m_V \to 9.28666$$

(c) Distance the star appears to be at:

$$d \to 10^{\frac{m_V - M_V + 5}{5}}\ \mathrm{pc} \;/.\; \{M_V \to -1.1,\ m_V \to 9.28666\}$$

$$d \to 1194.9\ \mathrm{pc}$$

Big difference!
Percentage error:

$$\frac{1194.9 - 720}{720} \times 100$$

$$65.9583$$

66%!!!!!!!

2. C&O Problem 12.10

Jeans Length for 1500 $M_\odot$:

$$R_J \to \left(\frac{3\,M}{4\,\pi\,\rho_0}\right)^{1/3} \mathrm{cm} \;/.\; \left\{M \to 1500 \times 1.989\,10^{33},\ \rho_0 \to 8.4\,10^{-22}\right\}$$

$$R_J \to 9.46497 \times 10^{18}\ \mathrm{cm}$$

or 6.3×10^4 AU or 3.067 pc.

Jeans Length for 8 $M_\odot$:

$$R_J \to \left(\frac{3\,M}{4\,\pi\,\rho_0}\right)^{1/3} \text{cm} \;/.\; \left\{M \to 8\times 1.989\;10^{33}\,,\; \rho_0 \to 3\;10^{-20}\right\}$$

$$R_J \to 5.02156\times 10^{17}\ \text{cm}$$

or 3.4 × 10^4 AU or 0.163 pc.

In[2]:=
$$R_J \to \left(\frac{15\,k\,T}{4\,\pi\,G\,\mu\,m_H\,\rho_0}\right)^{1/2} \text{cm} \;/.\; \left\{M \to 8\times 1.989\;10^{33}\,,\; k \to 1.381\;10^{-16},\; T \to 10,\; \mu \to 2,\; G \to 6.673\;10^{-8},\; m_H \to 1.673\;10^{-24},\; \rho_0 \to 3\;10^{-20}\right\}$$

Out[2]=
$$R_J \to 4.96082\times 10^{17}\ \text{cm}$$

3. C&O Problem 13.1

(a) Subtracting the times between the evolutionary points labelled yields:

Points	Time (Myr)	% of MS lifetime
1 to 2	92.9357	
2 to 3	1.5234	1.64
3 to 4	0.1144	0.12
4 to 5	0.3483	0.37
5 to 6	0.2890	0.31
6 to 7	4.1727	4.5
7 to 8	1.5045	1.6
8 to 9	6.320	6.8
9 to 10	1.246	1.3

(b) The Hertzsprung gap lies between point 4, where the Schoenberg-Chandrasekhar mass is reached (see pg. 458), and point 5, where the RGB begins. From (a), the time for this is 0.37% of the MS lifetime.

(c) The blueward portion of the horizontal branch (HB) is between points 7 and 8 and takes 1.6% of the MS lifetime.

(d) The redward portion of the HB is between points 8 and 9 and takes 6.8 % of the MS lifetime.

4. C&O Problem 13.2

The Kelvin-Helmholtz timescale is given by

$$t_{KH} \sim \frac{\mathcal{E}_{grav}}{L} \sim \frac{G\,M^2}{R\,L}$$

A 5 $M_\odot$ subgiant star is at point 4 on the HR diagram, with log $T_{\text{eff}} \approx$ 4.1 and log $L/L_\odot \approx$ 3.2. From these numbers, we can calculate the radius of the star:

$$L = 4\,\pi\,R^2\,\sigma\,T_{eff}^4$$

$$R \to \left(\frac{L}{4\,\pi\,\sigma\,T_{eff}^4}\right)^{1/2} \text{cm} \;/.\; \left\{L \to 10^{3.2}\;3.839\;10^{33},\; T_{eff} \to 10^{4.1},\; \sigma \to 5.67\;10^{-5}\right\}$$

$$R \to 5.83059\times 10^{11}\ \text{cm}$$

Thus R≈ 6 × 10^{11} cm.

$$t_{KH} \to \frac{G\,M^2}{R\,L}\,s\ /.\ \left\{G \to 6.67\,10^{-8},\ M \to 5 \times 1.989\,10^{33},\ R \to 5.83\,10^{11},\ L \to 10^{3.2}\,3.839\,10^{33}\right\}$$

$$t_{KH} \to 1.85973 \times 10^{12}\ s$$

or 59000 yrs, which is about 1/6 of the time between points 4 and 5.

5. C&O Problem 13.11

(a) The MS lifetime of a 0.8 $M_\odot$ star is considerably longer than the age of the universe; hence detailed study of low-mass star post-MS evolution is useless because such stars have never evolved beyond the MS, and never will in the foreseeable future of mankind.
(b) No. See (a).

6. Planetary Nebula & Supernova Remnants

Conditions needed to produce the glowing ring:
(i) The gas shell must be optically thin. Otherwise, the shell would look like an opaque disk, not a ring - so we wouldn't be able to see the central star through the shell. We see the outer edge of the shell most clearly because lines of sight towards the 'limb' of the shell pass through thicker layers of gas than lines of sight towards the shell's center. A thicker layer of gas implies more atoms fluorescing along the line of sight (as described in ii) below); this makes the shell's edges appear brighter than other regions of the shell. Note that the edge does not need to be optically thick (τ>1) - even from the viewpoint of an Earthbound observer - to appear brigther than the rest of the shell.
(ii) Since the shell is optically thin, the only mechanism that can produce the glow is line radiation, or photons produced when excited electrons in the shell's atoms fall back down to lower energy states. In order for the shell to glow, some mechanism must be exciting the electrons to higher energy states.
If the central star is hot enough, it may be producing enough photons of high enough energies to excited many of the elctrons and cause the glow. This is the mechanism behind planetary nebulae, whose central white dwarfs typically have temperatures of several times 10^4 K.
Alternatively, the gas shell may be expanding or moving outwards so fast that it produces shock fronts in the interstellar gas it encounters. This can heat the interstellar gas enough to excited the interstellar atoms and cause them to radiate. Since the shock fronts are usually close to the gas shell, it appears as if the shell is glowing. This is the mechanism behind some supernova remnants.