AY102: HOMEWORK 2

DUE: END OF DAY, 29 JANUARY 2016 (FRIDAY). TA’S MAILBOX.

[1] Spectroscopic Terms. Consider an unexcited Nitrogen (1s^22s^22p^3) atom. Show that there are only 20 independent states which map to 2P, 2D and 4S spectroscopic terms. Use the attached Grotrian diagram for Nitrogen\(^1\) and using the selection rules justify the permitted lines. \(\text{15 points}\)

[2] Hund’s rule. Look at the Grotrian diagram for the ground states of B(2P\(_{1/2}\)), Se(3P\(_{2}\)) and Ta(4F\(_{3/2}\)). Do these the terms follow Hund’s rule? \(\text{10 points}\)

[3] Pure Scattering. Here is a toy model for the Sun: only hydrogen (ionized, naturally) and with constant density. A photon is emitted in the core of the sun. Assuming only Thompson scattering how long will it take for the photon to reach the surface? \(\text{5 points}\)

[4] HII region. Consider a spherical HII region (pure Hydrogen) electron density, \(n_e\) and radius \(R\). The source function is provided by free-free emission (which we will discuss in class this week). The goal is to compute the flux from this sphere. s temperature instead of specific intensity (relating to the two via Rayleigh-Jeans formula).

Assume a constant source function and instead of intensity use brightness temperature. We will assume that the primary optical depth is primarily provided by free-free absorption

\[
\tau_{ff}(\nu) = 8.23 \times 10^{-2} T_e^{-1.35} \nu^{-2.1} EM
\]

where \(T_e\) is the temperature of the electrons (plasma), \(\nu\) is the frequency in GHz and EM is the emission measure, the line-of-sight integral, \(\int_0^l n_e^2 dl\) and the units\(^2\) are \(\text{cm}^{-2} \text{pc}\). HII regions are essentially thermostats with \(T_e \approx 8,000 \text{K}\).

Determine the output flux, by carrying out an exact integration of the equation of radiative transfer. The flux is clearly frequency dependent. Using the following parameters \(n_e = 10^4 \text{cm}^{-3}\), \(R = 0.5 \text{pc}\) and distance to the source, 1 kpc. Plot the spectral flux density from this source (units: Jansky) from say, 100 MHz to 100 GHz. Reflect upon the figure and then figure out whether there is a simpler way to have arrived at the asymptotic values without all this heavy duty algebra?

\(^1\text{http://hyperphysics.phy-astr.gsu.edu/hbase/atomic/nitrogenlev.html}\) Also attached for your convenience

\(^2\text{Welcome to Astronomy}\)
[5] **Hyperfine Lines.** The most famous hyperfine line is the 21-cm line of Hydrogen. The use of this line revolutionized the study of atomic medium in our own Galaxy, the rotation curves of galaxies (leading to the famous Tully-Fisher relation; first hint for the existence of dark matter). Please survey the literature (using ADS or Google) what other hyperfine lines have been detected by astronomers (name of line, frequency, name of mission).

5 points

---

**Figure 1.** Grotrian Diagram of N I.
Motivation

When I was a student I was, at times, frankly, not motivated to solve problem sets. I thought a lot about it and I now realize that I get engaged only when I am motivated. So let me motivate you!

It is and has been view that you ONLY learn when you understand concepts entirely by yourself. Concepts cannot be understood by listening to a lecture nor reading a book (they provide a starting point). Concepts can be understood only with concrete examples that you work through. In fact, the depth of understanding is directly related to how many concrete examples you have actually solved or thought three. So my first statement is: homework is the first opportunity for you to understand what you have heard in lectures and read in the book.

Next, homework problems become more interesting if you understand why a given problem is interesting. It helps if the problem is not a purely “toy” question. After all, you spent most of your schooling in formal education and probably are now bored with that approach.

[1] Spectral lines – atomic, molecular and inner-shell lines – are the primary diagnostics of the interstellar medium. As such understanding the physical basis of these lines is an essential requirement for a scholar in this field. While the quantum solution for hydrogen atom is elegant that for multi-electrons is complex (and naturally so). Thanks to L-S coupling you can reduce the enormous complexity of multi-electron atoms, at least for ground and near ground state transitions (optical, UV), to a model in which the multi-electron configuration is replaced by a single electron model but with \( L \) and \( S \) and their interactions. I realize that this is a difficult subject (but think of the gains from the last few lectures: you can now understand the periodic table better and you have some sense of the origin of spectral lines).

Research consists of mainly slogging through and understanding difficult concepts. There is no simple substitution for this. In the class I went through 2p2. I suggest that you review the 2p2 case and then proceed to the homework. It is a bit longer but at the end you will be slightly ahead in understanding the basics of atomic line spectroscopy.

[2] This is a simple problem and designed to make you familiar with the energy placement of the ground states. The curious student will attempt to understand the physical basis of Hund’s rule (by consulting, for example, http://hyperphysics.phy-astr.gsu.edu/hbase/atomic/hund.html#c4).

[3] This is a simple problem designed to reinforce the fundamental connection between optical depth and diffusion. I am sorry to say that the toy model is not even an ordinary toy but a ridiculous toy model.

[4] This is actually a nice problem. The problem can be solved analytically (with some work, though). The fun part is really plotting the spectral flux density. We will review
examples in our Galaxy and that is when you will see that your homework is actually allowing some inferences of measurements of objects in our Galaxy.

[5] Actually, I do not know much about new developments. I am using your enthusiasm and superior Google skills to educate me. In fact, one of the greatest advantages of teaching a course is that the instructor also benefits. For example, getting ready for this class made me ask the simple question: with all these new fabulous developments in radio astronomy are there some hyperfine lines that were in the past thought to be impossible to detect now detectable?