# Ay126: Homework 4

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**Notes:**

*The parameters of the WIM that you should adopt for this and other problems is given in §A.*

[**1**] **Sky brightness.** Search literature for the sky brightness in the $R$ band (new moon, zenith) at Mauna Kea and Paranal. The usual units are magnitude per square arcseconds. Compute the R-band sky brightness but expressed in Rayleighs[1]. [10 points]

Say, you have a narrow band imager (Fabry-Perot) at Keck. Compute the $5\sigma$ limit for the intensity in Rayleigh units that you would achieve for an hour of integration time, assuming a beam of radius 3 arcseconds in a band of 1 Å somewhere in the R band. [10points]

[**2**] **The Immediate Present: A Galactic FRB.** Just over a year ago, at 2020-04-28 14:34:33 CHIME telescope reported [ATEL 13681) a 5-ms wide burst with a dispersion measure of $332.8\,\mathrm{cm^{-3}\,pc}$ towards a soft gamma-ray repeater (SGR) 1951+2154. The arrival time is "topocentric"[2] and referenced to 400 MHz. The event was detected in a distant side-lobe and so the flux is not well known. Fortunately, Bochenek detected the same burst with his STARE2 project which consists of simple dipoles at several stations (Barstow, CA; Big Pine, CA; Delta, UT) operating in the 1.4 GHz band. The fluence (the product of flux density and pulse width) is properly measured by STARE2 and is found to be $1.5 \times 10^6\,\mathrm{Jy\,ms}$ (ATEL 13684). Soon thereafter the Chinese large radio telescope FAST detected a tiny pulse (fluence of 60 mJy ms) in the 1.25-GHz band and reported a Rotation Measure of $112.3\,\mathrm{rad\,m^{-2}}$. The magnetar is located in the Galactic plane and at the center of SNR G57.2+0.8. The distance to the magnetar is not known but reasonably argued (on the basis of association with its supernovae remnant etc) as being at $d = 10\,\mathrm{kpc}$.

**A.** Using the simple WIM model estimate the expected DM. If the mean magnetic field strength is $6\,\mu\mathrm{G}$ what is the maximum rotation measure you would expect. Can you plau-

[1] In astronomy, a Raleigh is a unit of intensity and is equal to $10^6/(4\pi)\,\mathrm{photons\,cm^{-2}\,s^{-1}\,sterad^{-1}}$

[2] which means the time measured at the surface – a fancy way of saying that the time was measured at the telescope

sibly account for the low RM? [10 points]

**B.** What is the expected difference in time of arrival between 1.4 GHz and 0.4 GHz. [Bonus: figure out the topocentric arrival time difference!]. [10 points]

As an exercise in pedagogy, we will assume that the source is much closer, say 3 kpc. In this case, our simple WIM model would predict a DM contribution of only $225\,\mathrm{cm}^{-3}\,\mathrm{pc}$. In this model, the difference is due to an intervening cloud (or the shell of the SNR etc). We will assume that this intervening cloud is spherical, diameter, $l$ and has a temperature of 8,000 K.

**C.** Compute the mean emission measure of the cloud as a function of $l$. With those in hand, compute the free-free optical depth at 0.5 GHz and 1.4 GHz, the integrated H$\alpha$ emission in Rayleigh units and the angular size of the cloud. Plot these quantities for $l$ ranging from 0.01 pc to 1 pc (and then think). [20 points]

**D.** Redo [C] for $d = 1\,\mathrm{kpc}$. [10 points]

[**3**] **Your Future: A Lunar Radio Telescope.** Enthused by Ay126 you decide to dedicate your life to studying the Universe at the lowest radio frequencies. The band below 10 MHz is routinely blocked by the ionosphere. So you are forced to a space mission. You consult Wikipedia and discover that the Earth is a powerful source of "Auroral kilometric radiation" (AKR; radiating a power 1–10 megawatts in the 50–500 kHz range).[3] So you are forced to locate your proposed telescope on the far side of the Moon ("Lunar Far-side Radio Observatory" or LFRO).

**A.** What is (low) frequency, $\nu_1$, below which celestial radiation (as in from other bodies or ISM beyond the solar system) will not reach your facility? [10 points]

**B.** What is the free-free optical depth at 1 MHz towards $b = 90°$? [10 points]

**C.** Consider a hot white dwarf located at a distance of 100 pc. The white ionizes the surrounding CNM and produces a small HII region for which the mean electron density is $100\,\mathrm{cm}^{-3}$, temperature of 8,000 K and the radius is 0.05 pc. Plot the flux from this HII region as a function of frequency from $\nu_1$ to 10 MHz. [15 points]

**D.** What is the physical size of FLRO that is needed to minimally image the Orion nebula, at say, 1 MHz? [Bonus: what other parameter can be critical in determining the spatial resolution of the image?] [10 points]

[3] AKR is cyclotron emission of energetic electrons and located above the magnetic poles at about $3R_E$.

ps. If this question got you excited check out `https://ui.adsabs.harvard.edu/abs/2019arXiv191108649B/abstract`)

[**4: Cooling of WIM.**] Compute the power radiated by a unit volume of the WIM. Consider Fine structure line of C+, O+, free-free recombination bound-free and two-photon decay. [25 points]

## A The Warm Ionized Medium

We will assume the following parameters for the WIM: physical density, $n_e = 0.5\,\mathrm{cm}^{-3}$, filling factor of 0.15, vertical (exponential) scale height of 1 kpc, temperature of 8,000 K and ionization fraction of $> 0.9$. Total mass in the Galactic WIM is $1.1 \times 10^9\,M_\odot$.