

# Eclipses

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[7-April-2009]

*Transiting Planets*  
*Proceedings IAU Symposium No. 253, 2008*  
*eds. F. Pont et al.*

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DOI: 00.0000/X000000000000000X

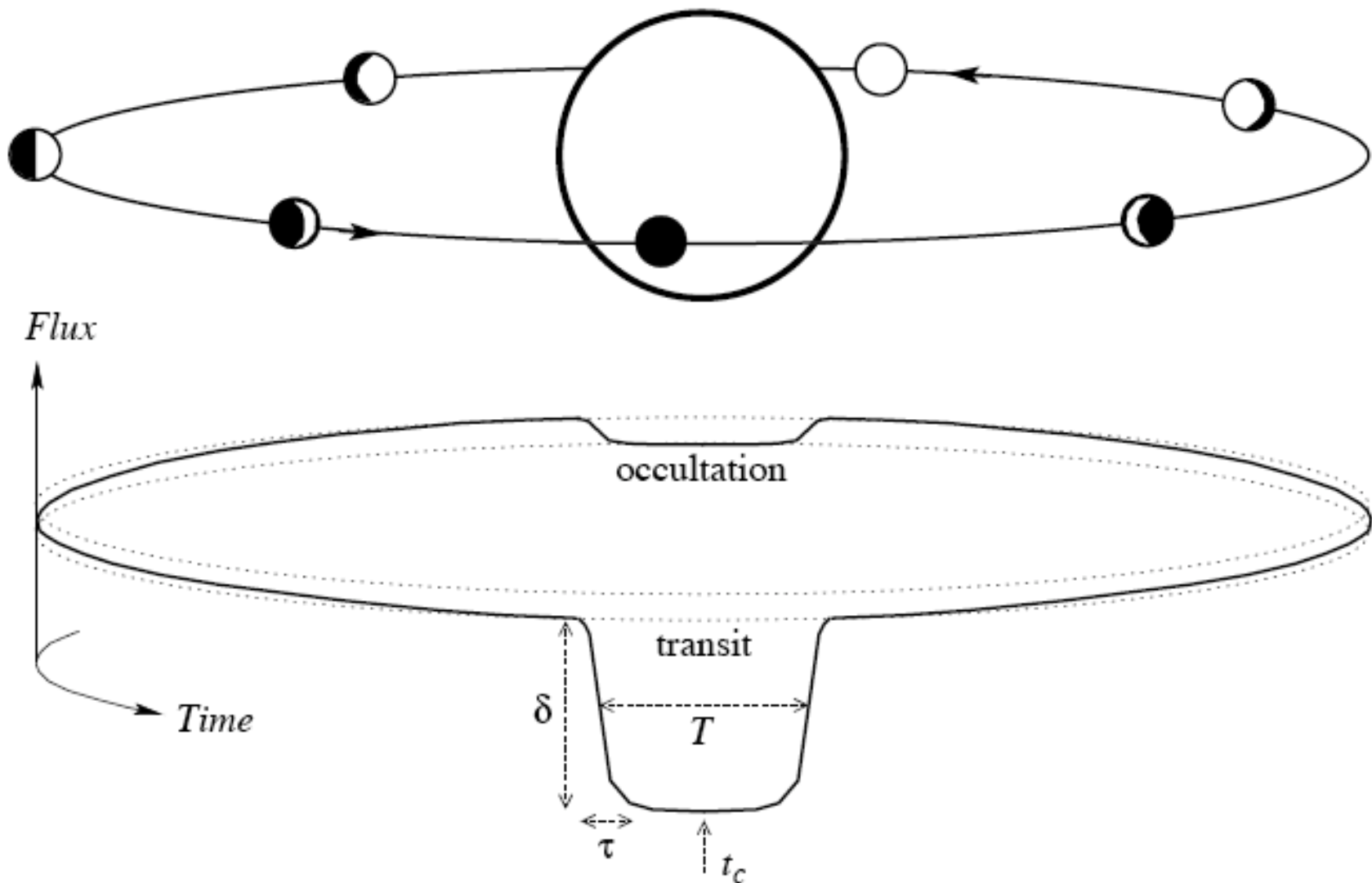
# Measuring accurate transit parameters

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**Abstract.** By observing the transits of exoplanets, one may determine many fundamental system parameters. I review current techniques and results for the parameters that can be measured with the greatest precision, specifically, the transit times, the planetary mass and radius, and the projected spin-orbit angle.

**Keywords.** planetary systems — eclipses — occultations — methods: data analysis



**Figure 1.** Illustration of transits and occultations. During a transit, the planet blocks a fraction of the starlight. Afterwards, the planet's brighter dayside comes into view and the total flux rises. The total flux drops again when the planet is occulted by the star.

**Table 1.** Properties that have been measured, or that might be measured in the future, through precise observations of transiting planets.

<b>Property</b>	<b>Refs.</b>	<b>Property</b>	<b>Refs.</b>
Orbital period	1,2	Planet-planet interactions (short-term)	19,20
Orbital inclination	1,2	Planet-planet interactions (long-term)	21,22
Planetary mass	1,2	Mutual orbital inclinations	20,23
Planetary radius	1,2	Planetary rings	24,25
Stellar obliquity	3,4	Satellites	9,24
Orbital eccentricity	5,6	Relativistic precession	26,27
Stellar limb darkening	7	Parallax effects	28, 29
Star spots	8,9	Apsidal motion constant	30
Thermal emission	5,10	Stellar differential rotation	31
Absorption spectrum	11,12	Oblateness and obliquity	32,33
Albedo	13,14	Variations in stellar radius	34
Phase function	15	Yarkovsky effect	35
Effective radiative time constant	16	Planetary wind speed	36
Trojan companions	17,18	Artificial planet-sized objects	37

## 2. Transit light curve parameters

In any discussion of measuring accurate transit parameters, the first question should be: what are those parameters? Ignoring limb darkening for the moment, the 4 basic observables are (with reference to Fig. 1) the mid-transit time  $t_c$ , the depth  $\delta$ , the total duration  $T$ , and the partial duration  $\tau$ . These observables can be translated into 3 dimensionless parameters describing the physical properties of the system:

$$\text{Radius ratio } R_p/R_s \approx \sqrt{\delta}, \quad (2.1)$$

$$\text{Impact parameter } b \approx 1 - \sqrt{\delta} \frac{T}{\tau}, \quad (2.2)$$

$$\text{Scaled stellar radius } R_s/a \approx \frac{\pi\sqrt{T\tau}}{\delta^{1/4}P} \left( \frac{1 + e \sin \omega}{\sqrt{1 - e^2}} \right), \quad (2.3)$$

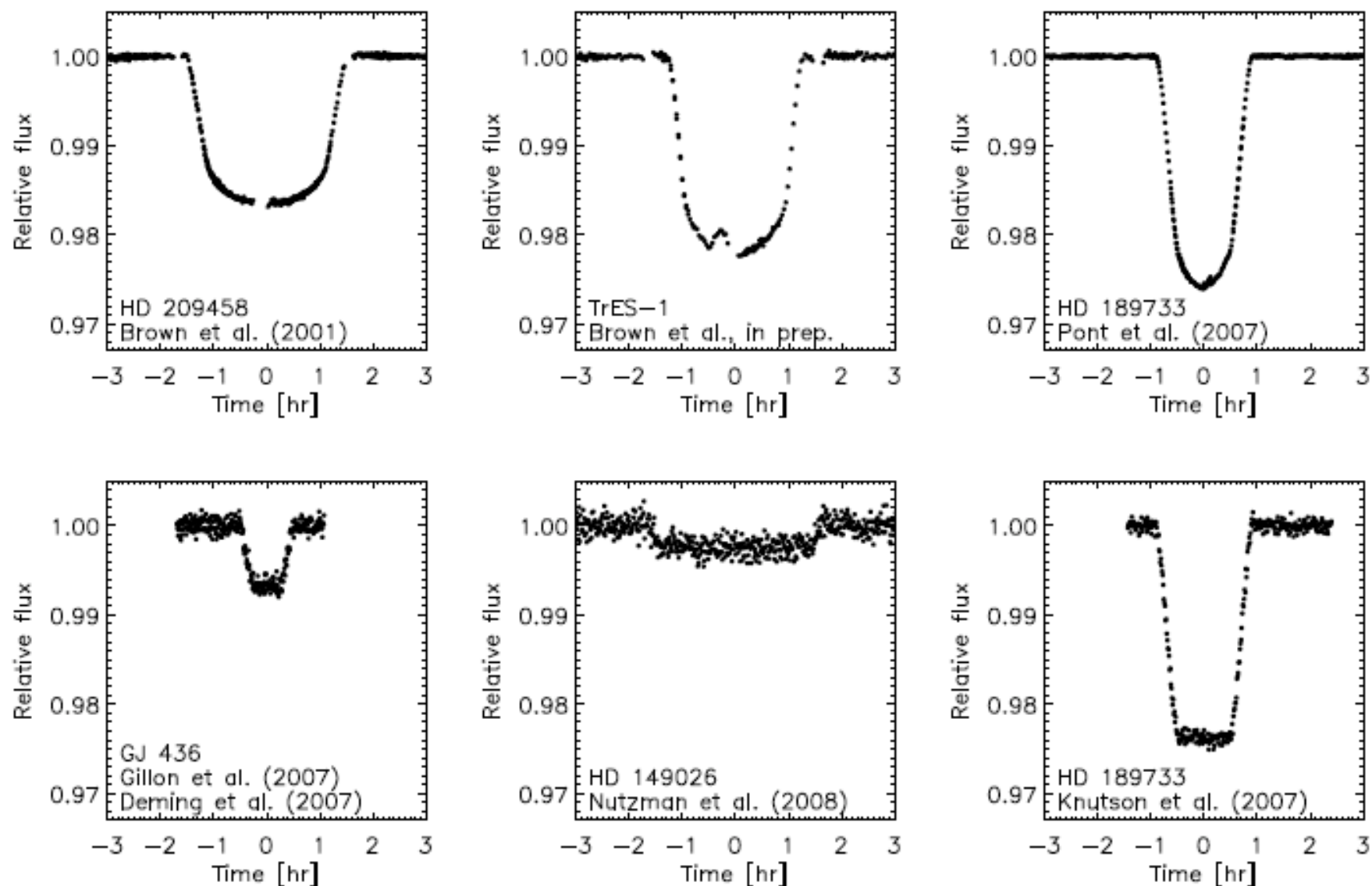
$$\text{Stellar mean density } \rho_s \approx \frac{3P}{\pi^2 G} \left( \frac{\sqrt{\delta}}{T\tau} \right)^{3/2} \left[ \frac{1 - e^2}{(1 + e \sin \omega)^2} \right]^{3/2} \quad (2.4)$$

$$\text{Planetary surface gravity } g_p \approx \frac{2\pi K_s}{P} \frac{\sqrt{1 - e^2}}{\delta (R_s/a)^2 \sin i}, \quad (2.5)$$

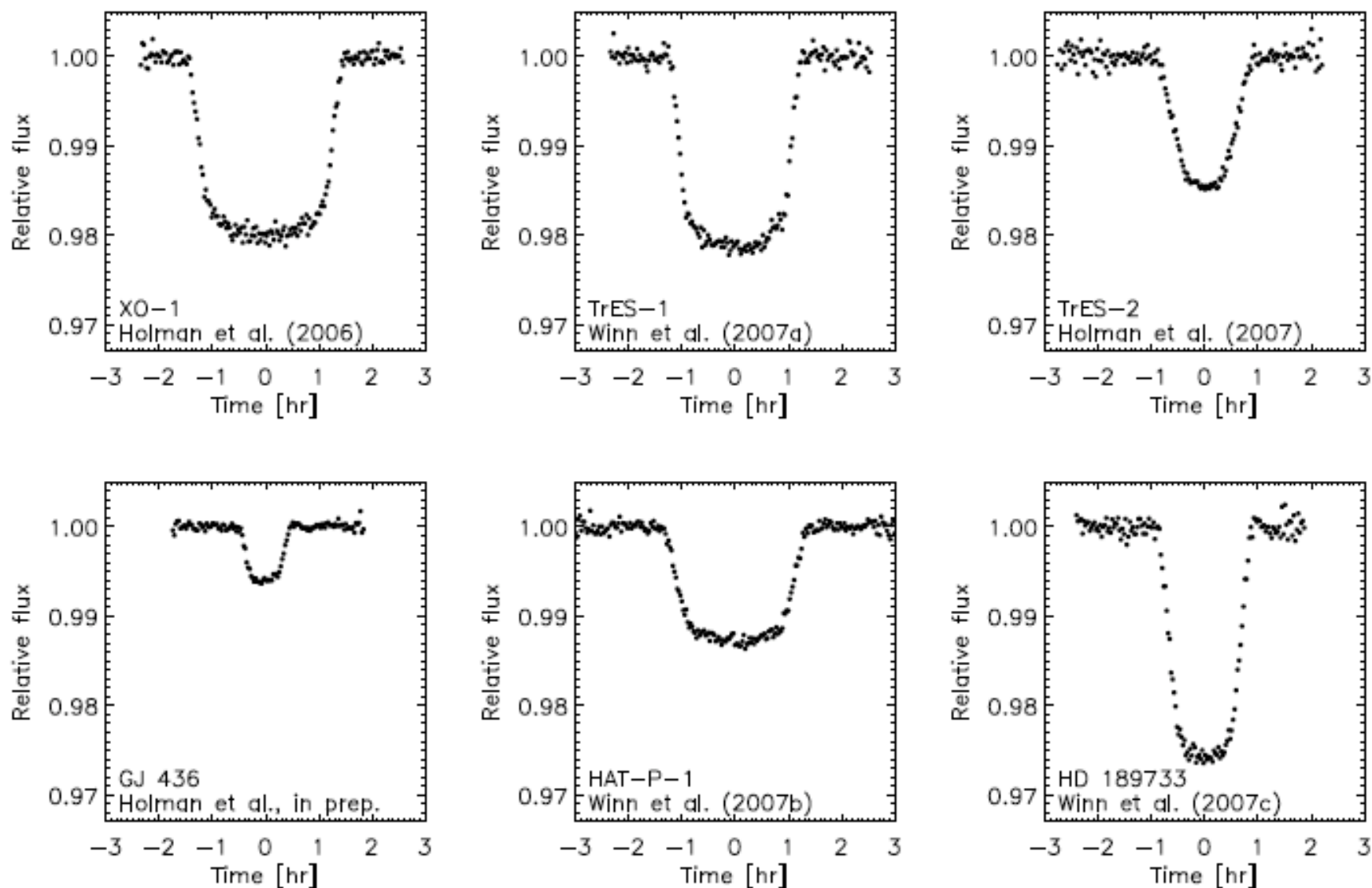
where  $K_s$  is the velocity semi-amplitude of the stellar radial-velocity signal, and  $i$  is the orbital inclination. The inclination can be written in terms of observables using

$$b = \frac{a \cos i}{R_s} \left( \frac{1 - e^2}{1 + e \sin \omega} \right). \quad (2.6)$$

Knowledge of the stellar mean density is helpful for pinning down the stellar properties. The photometrically-determined  $\rho_s$  is superior as a gravity indicator than the traditional  $\log g$  that is based on the widths of pressure-sensitive absorption lines (see, e.g., Sozzetti et al. 2007, Winn et al. 2008b). Recently, Torres et al. (2008) put this technique into practice for 23 transiting exoplanets, in the most homogeneous and complete analysis of transit data to date. A similar effort is underway by Southworth (2008). The relative immunity of  $g_p$  to systematic errors in the stellar properties suggests that when testing theoretical models of planetary structure, it would be wiser to compare theoretical and observed surface gravities, rather than the traditional comparison between theoretical and observed radii.

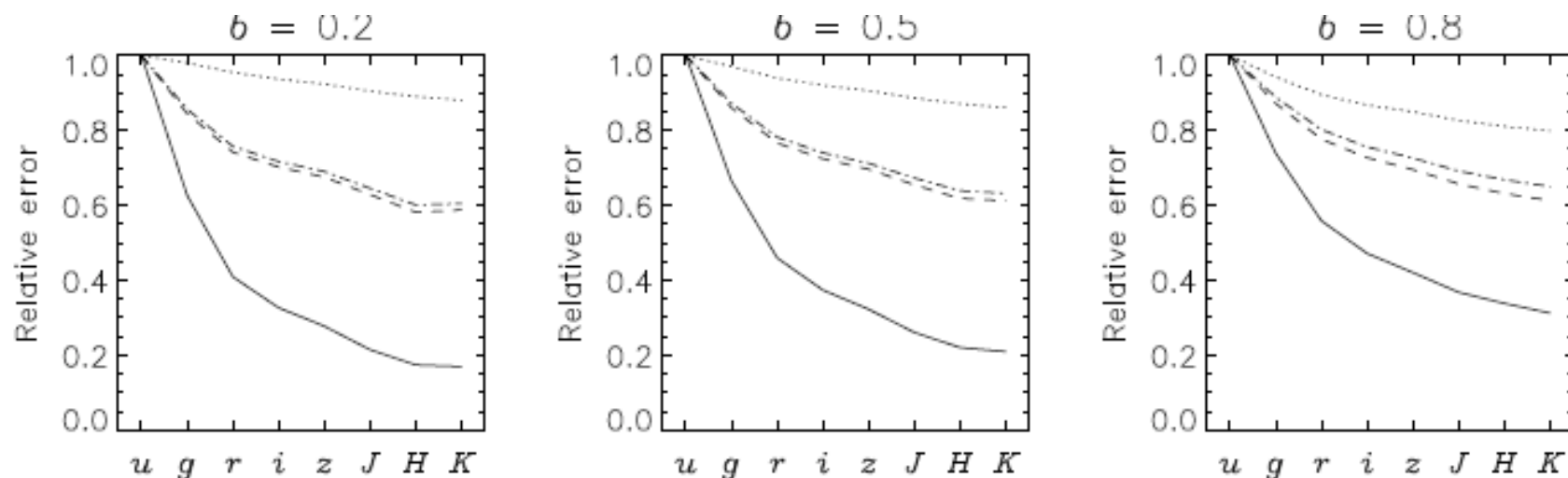


**Figure 2.** A gallery of transit light curves based on observations with spaceborne instruments. The upper 3 panels show optical data from the *Hubble Space Telescope* and the lower 3 panels show infrared data from the *Spitzer Space Telescope*.

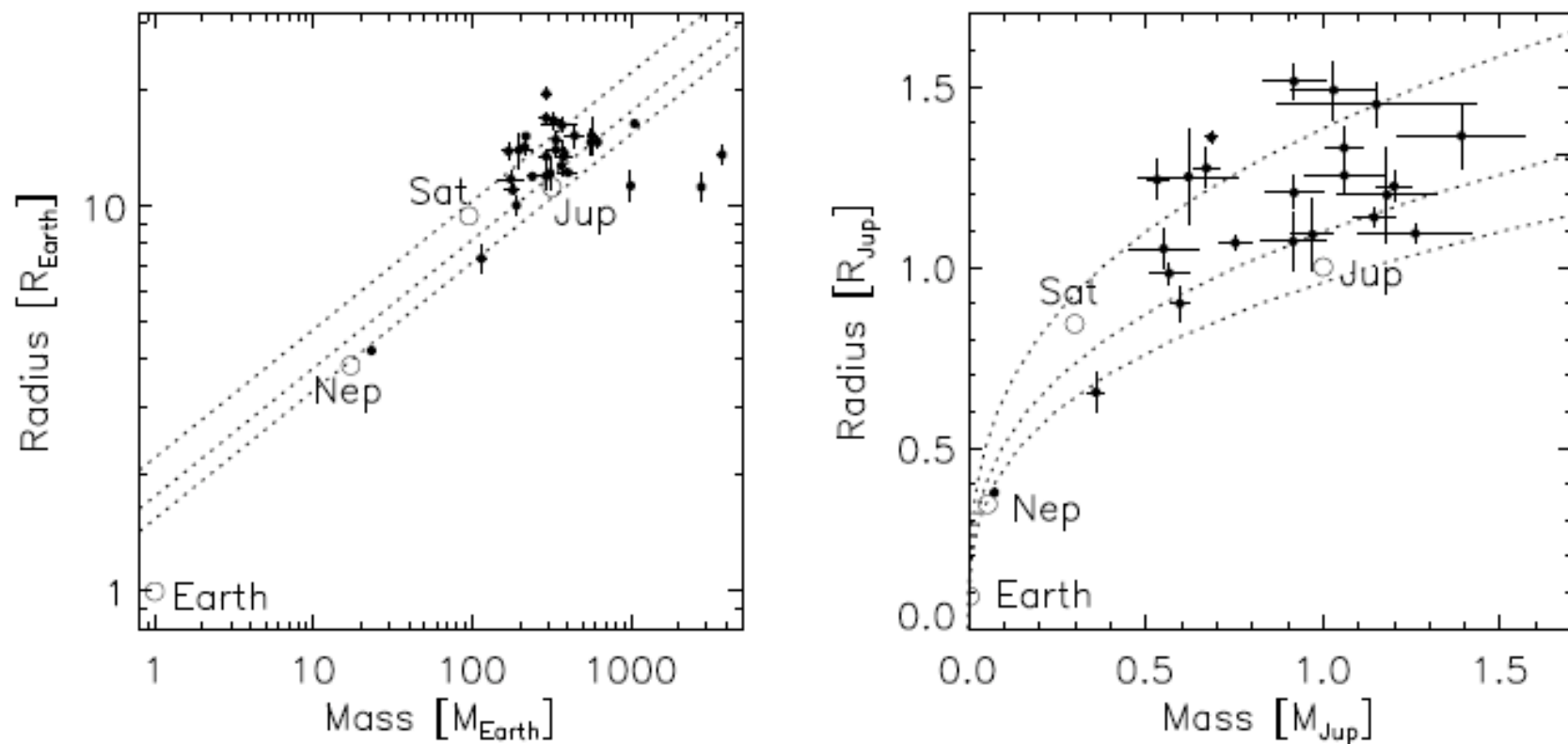


**Figure 3.** A gallery of transit light curves, based on observations with small ground-based telescopes. Each panel shows a time-averaged (100 s), composite light curve based on observations of multiple transits. The number of observed transits ranges from 2 (XO-1) to 11 (GJ 436).

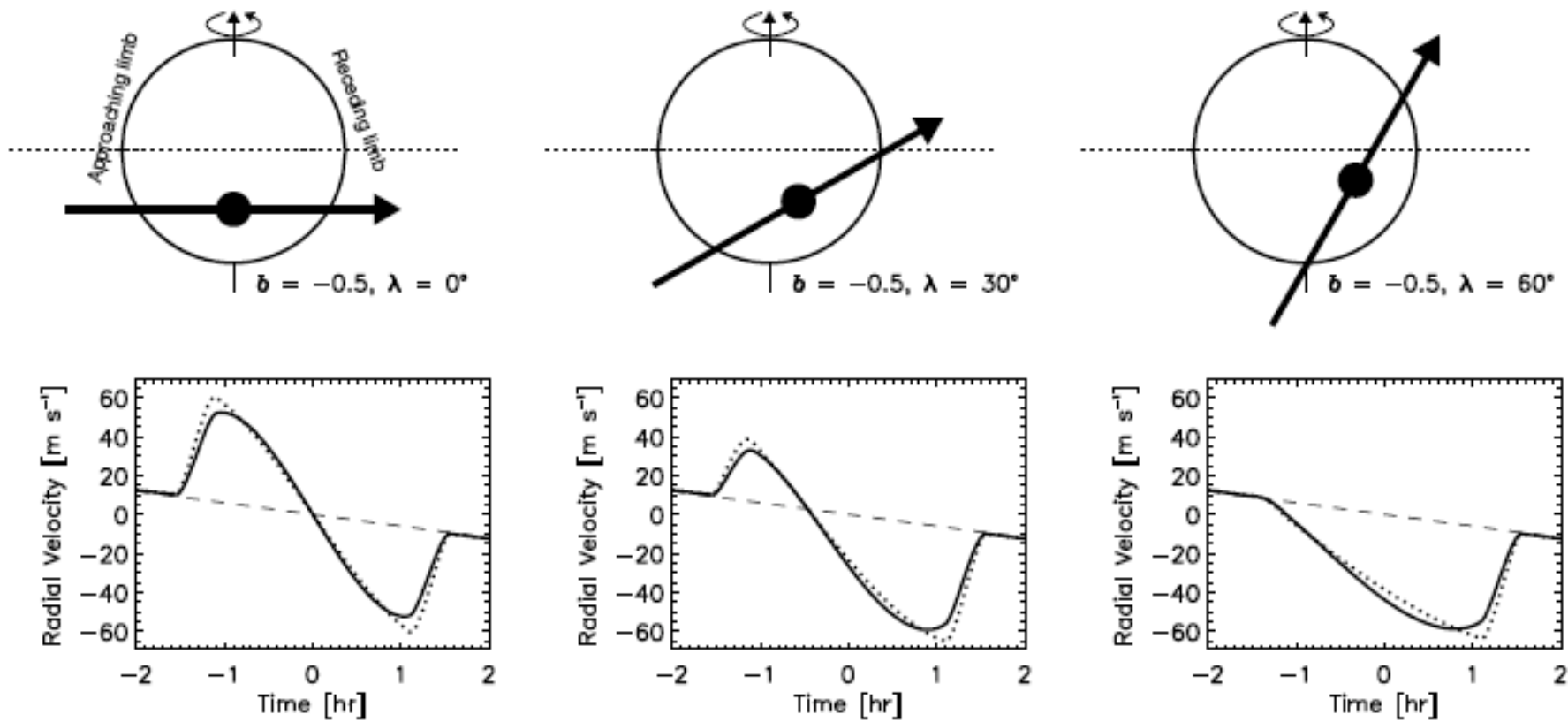
It is often advisable to use a long-wavelength bandpass, where extinction variations are smaller and the effects of stellar limb darkening are reduced. Smaller limb darkening leads to light curves with sharper corners and flatter bottoms, providing more statistical leverage on the parameters  $t_c$ ,  $R_p/R_s$ ,  $b$ , and  $R_s/a$ . To be concrete, Fig. 4 shows the



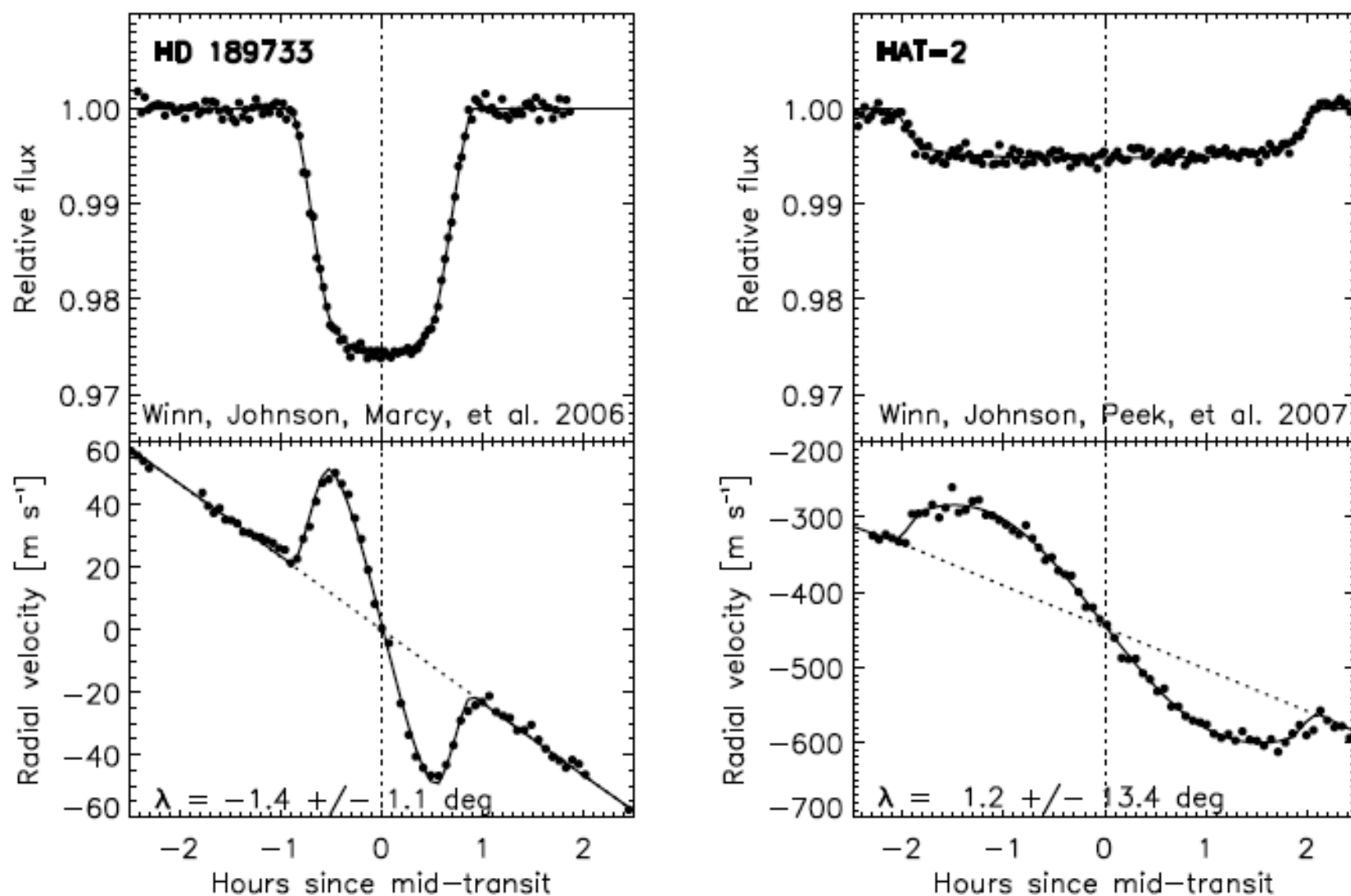
**Figure 4.** The effect of limb-darkening on parameter errors, based on calculations by Pál (2008). Imagine a planet with  $R_p/R_s = R_s/a = 0.1$  transits a Sun-like star with impact parameter  $b$ , and a light curve is obtained with  $1.3 \times 10^{-3}$  precision and 10 s cadence. Shown here is the relative error in the parameters  $R_p/R_s$  (solid),  $b$  (dashed),  $a/R_s$  (dash-dotted), and  $t_c$  (dotted), as a function of the observing bandpass. For redder bandpasses (to the right), the effect of limb-darkening is smaller and the parameter errors are decreased.



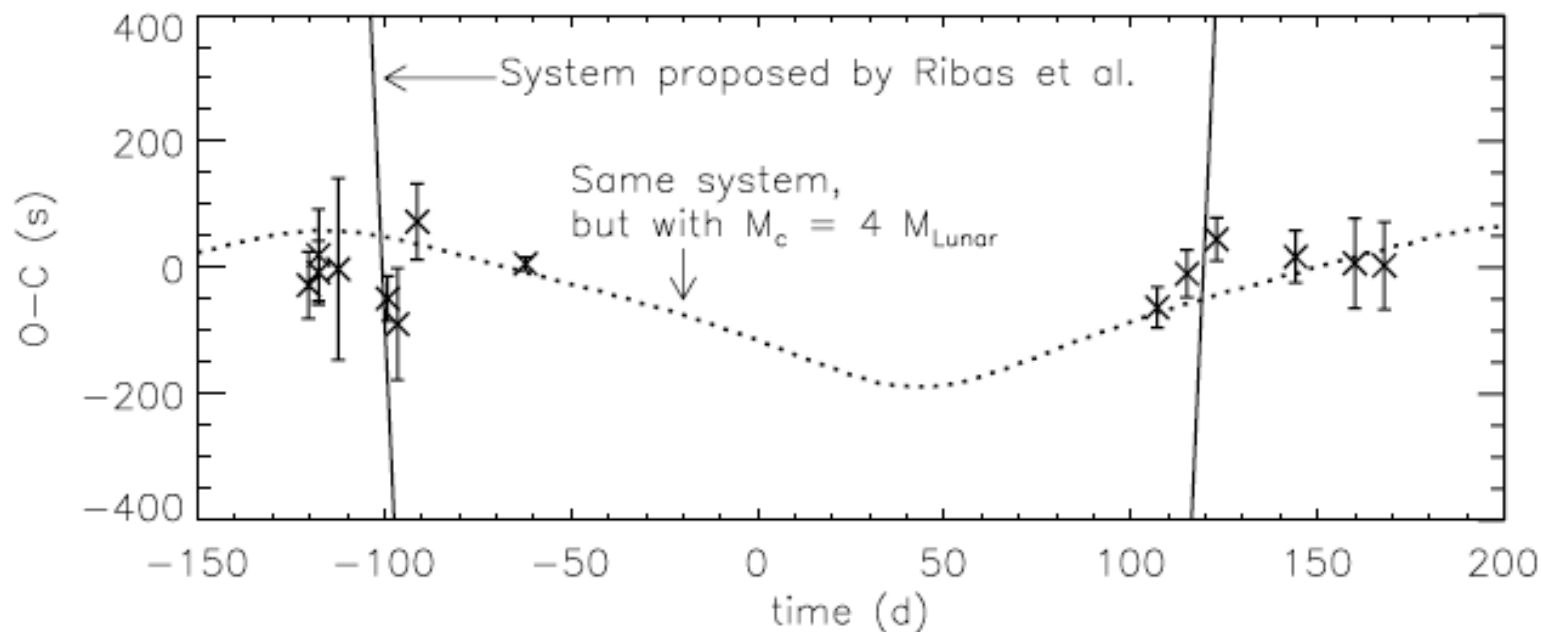
**Figure 5.** Masses and radii of transiting planets. The dotted lines are loci of constant mean density. Values for Jupiter, Saturn, Neptune, and Earth are also plotted, for comparison.



**Figure 7.** The Rossiter-McLaughlin (RM) effect as an “anomalous” Doppler shift. Top: three possible transit geometries that produce identical light curves, but differ in spin-orbit alignment. Bottom: The corresponding radial-velocity signals. Dotted lines are for the idealized case of no limb darkening; solid lines include limb darkening.



**Figure 8.** Examples of RM data. The top panels show transit photometry, and the bottom panels show the corresponding radial-velocity waveform. The out-of-transit gradient in the radial velocity is due to the orbital motion of the star. The in-transit “blip” is the RM effect.



**Figure 6.** Transit timing residuals (observed – calculated) for GJ 436, along with theoretical variations that would be expected due to a second planet in a 2:1 resonance, with a mass of  $5 M_{\oplus}$  (solid line) and  $0.05 M_{\oplus}$  (dotted line). Calculations and figure by D. Fabrycky.

# Thursday

- Should be able to derive the 3+3 basic equations of transit light curves
- Understand the propagation of errors (relative sensitivity) for key derived quantities
- Review comprehensive projects
  - 47 Tuc HST project
  - Ground based project(s)
- Goal: Estimate the potential use of synoptic projects for transits