Order of magnitude estimations and occasional precision estimates

Learn to be a swash buckler. For the first foray into a problem an order of magnitude estimate is sufficient. This means, focusing on the exponent value and to zeroth order setting all mantissa[1] values to unity!

The principal stumbling block faced by tyros is the tendency to carry the mantissa to high precision. After all for most of your life in school and college you have been drilled of the importance of exact calculations and are consequently experiencing "fear of imprecision". Do it a few times to get over the fear. Aim to be a swash buckler. For the next level, approximate the mantissa to a number close to unity. However, do remember that a mistake in adding and subtracting the characteristics will cost you dearly. It is my observation that it will take many months of disciplined and sustained effort to get used to even the first-order level of swash buckling.

For rapid computation it is *essential* to have a comprehensive grasp of mathematical, physical and astronomical constants. Finally, it is efficient to arm yourself with already constructed formulae (which I call as "pre-computations").

Higher Precision. However, a higher precision is needed when you are considering whether an effect is observable or not. Another occasion is that you are listening to a seminar and the speaker proposes an explanation for an observation. It is fun and occasionally very productive to figure out whether the proposed model has observational consequence. If you can compute in real time you can impress your colleagues with your brilliance! So, remember to take a paper and pencil to all talks.

The minimum necessary mathematical constants for an observational astronomer are given in the accompanying Table. From this table, many other frequently needed constants ($\ln(2) = \ln(10)\log(2) = 0.69$, $1/\sqrt{2} = \sqrt{2}/2 = 0.707$, etc.) can be derived. The half-life period is $\ln(2)\tau \approx 0.69\tau$ where the radioactive material decays as $\exp(-\tau)$.

[1] as in value $= m \times 10^n$ where m is the mantissa (also "significand") and n is the exponent value or characteristic.

Table 1: Some useful mathematical values

name	value
π	3.1415
$\sqrt{2}$	1.414
$\sqrt{3}$	1.732
$\log(2)$	0.301
$\log(3)$	0.477
e	2.718
$\ln(10)$	2.306
FWHM (Gaussian)	$\sqrt{\ln(2^8)} \approx 2.35$

I recommend that the student commit to memory[2] the constants given in Table 1. Sometimes, stories help reconstruct numbers. A classical way to celebrate the amazing π is to eat a piece of pie on 14th March at 1500 hours. I sleep between 22 hours and 7 hours and 22/7 is sometimes an adequate approximation of π for calculations.

Log[3]. There are two reasons why astronomers must be able to compute the log of a number and equally to compute the number, given the log. The first is that astronomers use magnitudes which are defined by $m = -2.5\log(f) + m_0$ where f is the measured flux and m_0, an offset, provides much fodder for older astronomers to impress neophytes. Next, many astronomical plots or tables have log of the physical quantities (e.g. mass-luminosity relation, table of absolute magnitude versus mass of star).

Fortunately, you need to remember only log(2) and log(3). The values for 4, 6, 8, 9 follow from these two. Likewise, $\log(5) = \log(10) - \log(2) = 0.7$. Noting that $10/\sqrt{2} \approx 7$ we derive a simple way to remember $\log(7) \approx 0.85$.

Magnitudes. To start with, since $\log(2.5) = \log(10/4) \approx 1 - 0.6 = 0.4$ it follows that one magnitude corresponds to a factor of 2.5. A source which doubles in brightness by a factor of two will appear $\approx 2.5 \times 0.3 = 0.75$ mag brighter.

Astronomers frequently quote symmetric errors in magnitude but it is not correct since the underlying flux measurement, f, is almost always Gaussian (and occasionally Poisson). However, when the magnitude errors are small the distribution is approximately Gaussian. Let the measured flux be $f_0 \pm \Delta f$. Then the uncertainty in the magnitude is

$$\Delta m = -2.5\log\left(1 \pm \frac{\Delta f}{f_0}\right) \approx \pm\frac{2.5}{\ln(10)}\frac{\Delta f}{f_0} = \pm 1.1\frac{\Delta f}{f_0} \tag{1}$$

[2] One way to commit to memory is to use the mantra approach: read aloud Table 1 every morning until you can recite the values.

[3] I use the convention of log and ln to mean logarithm to the base 10 red and to the base e, respectively.

where I have retained the first term of Taylor series expansion of $\ln(1+x)$. Thus, for high signal-to-ratio (SNR) measurements, the uncertainty in the magnitude is equal to the relative precision of the measurement or equivalently to the inverse of the SNR.

Square Root. Occasionally it helps to know the square root of a number. You can use Newton's method:

$$\sqrt{n} = \frac{1}{2}\left(\frac{n}{a} + a\right) \tag{2}$$

where a is your guess value for $\sqrt{n}$. As an example, consider $n = 3$ and we set the guess value as 2. Then, $\sqrt{3} \approx 1/2(3/2 + 2) = 7/4 = 1.75$ (which can be compared with the more precise value given in Table 1).

Full Width At Half-Maximum. The half width half-maximum of a Gaussian profile, x, is given by $\exp(-x^2/2) = 1/2$. Thus, the full width at half-maximum is FWHM $= 2 \times \sqrt{2\ln(2)} = \sqrt{\ln(2^8)} = \sqrt{8 \times \ln(2)} = 2.35$.

Inflation Escalator. Our entire life which includes research funding is governed by finance (micro, macro, personal). The cost of research (basic material cost, energy, payroll) increases with inflation, at the very least. The doubling time for a fixed (true) cost item is $n_d = \ln(2)/\ln(1+p)$ where p is the inflation increase per year. For small p Taylor expansion can be profitably used and so

$$n_d = \frac{\ln(2)}{p(1-p/2)} \approx \frac{70}{P}\left(\frac{1}{1-p/2}\right) \approx \frac{70}{P}\left(1 + \frac{p}{2}\right) \text{ years.} \tag{3}$$

Here, $P = 100p$ and is the annual inflation increase ~~but~~ expressed as a percentage. I provided the second term since in financial matters precision matters! In most countries (excluding Japan and Zimbabwe) the inflation index is between 3% and 10%. Thus a ~~a~~ simple rule: $n_d \approx 71/P$ years.

Last Revised: January 27, 2020