

**AY 102/126 FINAL EXAM**

**Winter 2008**

Do exam during a continuous 3-hour period. If you need a little more time, take it and note on the front of exam how much time you used. Please do not work on exam for more than 4 hours.

**AY 102 : Open book, open notes, closed homework**

**AY 126 : Open personal notes, closed book, closed class handouts, closed homework**

**AY 102 & AY 126 : Answer all short questions (Section 1) in no more than a few sentences each. All problems in Section 1 count equally.**

**AY 102 & AY 126 : All problems in Section 2 count equally.**

**AY 102 : DO 4 OF THE 6 PROBLEMS IN SECTION 2**

**AY 126 : DO 5 OF THE 6 PROBLEMS IN SECTION 2**

[Section 1 is worth a total of 40 points. Each question in Section 2 is worth 20 points. No bonus points will be awarded.]

**PLEASE RETURN EXAM BY 5:00 PM ON WEDNESDAY MARCH 19, 2008,  
TO JUDITH MACK - ROOM 105 ROBINSON.**

**Useful Constants: (in cgs units)**

$$\mathbf{h} = 6.63 \times 10^{-27} \text{ erg s}$$

$$\mathbf{k} = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$

$$\sigma \text{ (Stefan-Boltzmann)} = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$$

$$\mathbf{c} = 3.00 \times 10^{10} \text{ cm/s}$$

$$\mathbf{G} \text{ (Gravitational constant)} = 6.7 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$$

$$\mathbf{m}_H = 1.66 \times 10^{-24} \text{ g}$$

$$\mathbf{1 \text{ eV}} = 1.6 \times 10^{-12} \text{ ergs}$$

$$\alpha_A = 4 \times 10^{-13} (10^4/T)^{1/2}$$

$$\alpha_B = 2.6 \times 10^{-13} (10^4/T)^{1/2}$$

**photo-ionization cross-section for H :  $a(\lambda) = 6 \times 10^{-18} (\lambda_o/\lambda)^3 \text{ cm}^2$  for  $\lambda < \lambda_o$**   
**where  $\lambda_o = 912\text{\AA}$**

$$\mathbf{1 L_\odot} = 3.9 \times 10^{33} \text{ ergs/sec}$$

$$\mathbf{1 M_\odot} = 2 \times 10^{33} \text{ gm}$$

$$\mathbf{1 \text{ pc}} = 3.1 \times 10^{18} \text{ cm}$$

$$\mathbf{1 \text{ AU}} = 3.1 \times 10^{18} \text{ cm}$$

## 1. Section 1

- 1 When describing the shape of a spectral line, what is the difference between a Lorentz profile, a Doppler profile, and a Voigt profile?
- 2 What are the primary components of interstellar dust and what is the typical range of dust grain sizes? How do we know this?
- 3 What is the observational evidence for the existence, direction and magnitude of interstellar magnetic fields?
- 4 The dominant component of molecular clouds is molecular hydrogen,  $\text{H}_2$ . Why are the the properties of these clouds not studied by measuring the emission/absorption of  $\text{H}_2$ ? How is the  $\text{CO}/\text{H}_2$  and the gas/dust ratio measured?
- 5 Describe simple observations you might use to determine the temperature and density in an HII region.
- 6 As a supernova evolves, a number of stages can be identified. Describe each stage briefly. In each phase, how do the radius and velocity of the remnant vary as a function of time?
- 7 What extra information can be derived by mapping a typical molecular cloud in  $^{12}\text{C}^{16}\text{O}$  (1–0) and at least one other isotopologue, such as  $^{13}\text{C}^{16}\text{O}$  (1–0), rather than just observing  $^{12}\text{C}^{16}\text{O}$  (1–0)?
- 8 What is the difference between a J-shock and a C-shock?
- 9 What effects does a protostellar outflow have on the evolution of the protostar and its environment?
- 10 Observations of regions such as the Eagle Nebula have shown that it is fairly common to find cores of (neutral) molecular gas embedded in HII regions. How can the existence of these objects be explained when we know that  $\geq 99\%$  of the gas in an HII region is ionized?

## 2. Section 2

- 1 The Jeans' criterion indicates the minimum size  $\Lambda_J$  a gas cloud must have if it is to collapse under its own self gravity.
  - a) Estimate the Jeans length,  $\Lambda_J$ , and corresponding Jeans mass  $M_J$  for (i) a dense H I cloud ( $n_H = 100 \text{ cm}^{-3}$ ,  $T = 80\text{K}$ ) and (ii) a molecular cloud core ( $n_H = 10^8 \text{ cm}^{-3}$ ,  $T = 150\text{K}$ ). Are the properties of these regions conducive to star formation?
  - b) Compare the free fall times for these two regions with cloud lifetimes and with the ambipolar diffusion timescales. What are the implications for star formation?
  - c) In the expanding universe, galaxy formation is thought to begin about 1 million years after the Big Bang, when  $T = 3000\text{K}$  and  $n_H = 3000 \text{ cm}^{-3}$ . Estimate the corresponding values of  $\Lambda_J$  and  $M_J$ .
- 2 It is generally believed that galaxies form in the early universe by collapse of protogalactic HI clouds and that the first star formation occurs when the clouds are still quite extended. For a spherically symmetric HI cloud with mass  $10^{11} M_\odot$ , outer radius of 10 kpc, and uniform density distribution:
  - a) Estimate the Lyman continuum emission rate (photons/sec) for a centrally located starburst with total luminosity of  $10^{12} L_\odot$ . [Hint: You don't need to assume a temperature, but state other simplifying assumptions.]
  - b) Estimate the size of the ionized Stromgren sphere (assume an average density of  $n_H = 100 \text{ cm}^{-3}$  and neglect dissociation of  $\text{H}_2$ ).
  - c) Estimate the fractional ionization (i.e.  $n_e / n_{HI}$ ) of the HI in the HII region at  $\sim 1/2$  the Stromgren radius.
  - d) Suppose the nucleus of the galaxy is enveloped in a cloud of dust at typical distance 50 pc. Using the starburst luminosity, estimate the dust temperature and the wavelength of peak dust emission.

- 3 For a 2-level atomic system with level separation of 1 eV, a spontaneous decay rate of  $10^{-4} \text{ sec}^{-1}$ ,  $\langle \sigma v \rangle_{ul} = 10^{-11} \text{ cm}^3 \text{ sec}^{-1}$ ,  $g_l = 1$ , and  $g_u = 3$ :
  - a) Derive expressions for the level population ratio ( $n_u/n_l$ ) and the radiative cooling rate (assume optically thin emission).
  - b) Estimate the critical density ( $n_{crit}$ ) of the transition.
  - c) For  $n_{atom} = 10^4 \text{ cm}^{-3}$ ,  $n_e = 10^6 \text{ cm}^{-3}$  and  $T_e = 10^4 \text{ K}$ , estimate the cooling rate (in  $\text{ergs s}^{-1} \text{ cm}^{-3}$ ) and excitation temperature.
  - d) What do you think the excitation temperature and cooling rates will be if  $n_e = 1000 \times n_{crit}$ ?
- 4 Observations suggest that it is over-simplistic to imagine that stars form from the free fall collapse of rotating cloud cores.
  - a) What are the major problems? Be as quantitative as possible.
  - b) Does the presence of a magnetic field help resolve these difficulties? Provide specific details of the effect of the magnetic field on a collapsing core.
  - c) If a spherical gas cloud with mass  $M$ , radius  $R$ , and uniform density, is supported against collapse by a magnetic field of strength  $B$ , derive an expression for the critical mass,  $M_{crit}$ , the largest mass that can be supported against collapse by field  $B$ . Estimate  $M_{crit}$  for a cloud of radius 3 pc, that is threaded by a typical interstellar magnetic field.

- 5 Consider a spherical molecular cloud with  $10^4 M_\odot$   $T = 15$  K, measured CO linewidth of 3 km/s and radius 10 pc. Inside this molecular cloud is a dense core with  $10 M_\odot$   $T = 10$  K, measured CO linewidth of 0.3 km/s and radius 0.1 pc. You may assume that both objects have uniform density and temperature. For both objects, answer the following questions:
  - a) What is the thermal linewidth for CO and H<sub>2</sub>, and how does this compare with the observed linewidth?
  - b) What linewidth would you predict for the H<sub>2</sub> molecule?
  - c) Are these objects gravitationally bound?
  - d) Estimate the average hydrogen column density, dust mass and extinction ( $A_V$ ). Can you measure the optical light from background star behind each object?
- 6 For the 21-cm transition of HI the A-coefficient is  $3 \times 10^{-15} \text{s}^{-1}$  and the collisional cross-section (collision with other H atoms) is  $3 \times 10^{-15} \text{cm}^2$ . The statistical weights for the ground ( $g_1$ ) and excited ( $g_2$ ) states are respectively 1 and 3. Consider an atomic cloud with  $n_{\text{H}} = 40 \text{ cm}^{-3}$  and  $T = 80$  K.
  - a) Estimate  $T_c$ , the mean time between collisions for a given H atom. Compare  $T_c$  with the mean time for radiative decay.
  - b) Estimate the number density of H atoms in the excited state.
  - c) Estimate the rate of cooling in the 21-cm HI line..