Interstellar Dust Grains I

See Kwok chapters 10-13.

1. Interstellar extintion

See Kwok Chapter 10.1.

Let us define the *color excess*

$$E_{B-V} \equiv (B-V) - (B-V)_0$$

where $(B-V)$ is the observed color index and $(B-V)_0$ is the intrinsic values of the color index (see Kwok Eq 3.5).

The *total extinction* A_λ is the increase in apparent magnitude of a background object (e.g. a star) at wavelength λ.

$$m_\lambda = M_\lambda + 5\log d - 5 + A_\lambda$$

$$A_\lambda = (m_\lambda - M_\lambda) - (5\log d - 5)$$

where m_λ is the apparent magnitude of the star, M_λ is the absolute magnitude of the star (at wavelength λ) and d is the distance in pc.

The ratio of the total extinction to the color excess is given by.

$$R_V = \frac{A_V}{E_{B-V}}$$

R_V is dependent on the composition and size of the dust grains.

$$\frac{A_\lambda - A_V}{A_B - A_V} = \frac{E_{\lambda-V}}{E_{B-V}} = \frac{A_\lambda}{E_{B-V}} - R_V$$

By assuming that $A_\lambda \to 0$ as $\lambda \to \infty$, we get,

$$R_V = -\left[\frac{E_{\lambda-V}}{E_{B-V}}\right]_{\lambda\to\infty}$$

One can extrapolate the observed mean extinction curve to long wavelengths to derive R_V. Measurements of R_V made in this way typically find values of $R_V \sim 3$ in the diffuse ISM and $R_V \sim 5$ in dense regions within molecular clouds.

2. Dust Absorption and Scattering

See Kwok chapter 10.2.

In the case that the wavelength of light is much smaller than a spherical dust grain of radius a, the absorption coefficient is given by,

$$\kappa_\nu = \pi a^2 n_d$$

However, dust grains are have size $a \sim 1\ \mu$m or smaller, so the $a \gg \lambda$ approximation is not valid. Instead, we introduce a dimensionless number $Q_\nu(a)$ called the *extinction coefficient* such that:

$$\kappa_\nu = \pi Q_\nu(a) a^2 n_d$$

In general, extinction can be caused by absorption and scattering, and one can break up Q into two components,

$$Q_{\text{ext}} = Q_{\text{abs}} + Q_{\text{sca}}$$

In principle the absorption and scattering constants can be measured in a lab for different materials, but in practice this is hard because of the uncertain composition of interstellar dust grains. Similarly, the extinction coefficients can be sovled using Maxwell's equations with appropriate boundary conditions, but this again suffers from the same uncertainties.

The approximation usually adopted for the absorption and emission of dust grains is,

$$Q \sim Q_0 \left(\frac{\lambda}{\lambda_0}\right)^{-\alpha}$$

where α has a value of $\sim 1 - 2$ depending on the grain composition.

The optical depth due to dust absorption along the line of sight is

$$\tau = \int \kappa_\nu ds = \int \pi a^2 Q_\nu n_d ds$$

Expressed in terms of V-band extinction,

$$A_V = 1.086 \pi a^2 Q_V N_d$$

where $A_\lambda = 1.086 \tau_\lambda$ and $N_d = \int n_d ds$.

If one assumes a typical dust to gas mass ratio of $\psi = 10^{-2}$, and assuming that $Q_V \sim 2$ and $a = 0.1\ \mu$m, then one derives this handy conversion:

$$A_V \simeq 10^{-21} N_H$$

3. Dust Emission

In thermodynamic equilibrium, the emission coefficient is given by Kirchoff's law:

$$\frac{j_\nu}{\kappa_\nu} = B_\nu$$

$$j_\nu = (\pi a^2 Q_\nu) n_d B_\nu(T_d)$$

where T_d is the dust temperature and j_ν has units of [erg s^{-1} cm^{-3} sr^{-1} Hz^{-1}].

In the opically thin limit, the observed flux for a spherical cloud of radius R is given by,

$$F_\nu(D) = \frac{4\pi j_\nu}{4\pi D^2}\frac{4\pi R^3}{3}$$

$$F_\nu = \frac{\psi M_{\rm gas} Q_\nu B_\nu(T_d)}{\frac{4}{3} a \rho_s D^2}$$

where ρ_s is the density of the dust grain and F_ν has units of [erg s^{-1} cm^{-2} Hz^{-1}].

Alternatively, one may write that the intensity is given by

$$S_\nu = \Omega B_\nu(T_d) \kappa_\nu m_H N_H$$

where Ω is the beam size, m_H is the mass of hydrogen and N_H is the hydrogen column density.

If one knows the shape of the spectral energy distribion of dust emission, the (isothermal) dust temperature can be measured by taking the ratio of fluxes at two wavelengths.

$$\frac{S_{\nu_1}}{S_{\nu_2}} = \left(\frac{\nu_1}{\nu_2}\right)^{3+\alpha} \frac{\exp(h\nu_2/kT_d) - 1}{\exp(h\nu_1/kT_d) - 1}$$

After solving for T_d, one can then solve for N_H, assuming some dust emissivity and gas to dust ratio. If one had several measurements of the dust SED, then one can also solve for the shape (α) of the dust emission spectrum.

4. Heating and Cooling of Dust Grains

Let's consider the case of cloud of dust heated by a central star. In this case the heating rate (Γ) is given by

$$\Gamma = \int_0^\infty \kappa_\nu (4\pi J_\nu) d\nu$$

where J_ν is the mean intensity at the location of the grains.

Assuming that direct starlight is the dominant heating source, the mean intensity at radial distance r ($r \gg R_*$) is given by

$$J_\nu = \frac{1}{4}\left(\frac{R_*}{r}\right)^2 B_\nu(T_*)$$

Assuming that the dust grain cools by self-radiation, the cooling rate (Λ) is given by

$$\Lambda = \int_0^\infty 4\pi j_\nu d\nu$$

In thermal equilibrium, heating and cooling are balanced ($\Gamma = \Lambda$), so we find that

$$\int_0^\infty \frac{1}{4}\left(\frac{R_*}{r}\right)^2 B_\nu(T_*)(Q_\nu \pi a^2) d\nu = \int_0^\infty (Q_\nu \pi a^2) B_\nu(T_d) d\nu$$

If one assumes that the dust grains are large compared to the average wavelength of the stellar emission, then Q_ν can be taken outside of the integral on the left-hand side of the equation and $Q_{\rm abs} \sim 1$. In this case,

$$\frac{1}{4}\left(\frac{R_*}{r}\right)^2 \left(\frac{\sigma T_*^4}{\pi}\right) = \int_0^\infty Q_\nu B_\nu(T_d) d\nu$$

where σ is the Stephan-Boltzmann constant.

By substituting $Q_\nu = Q_0(\lambda/\lambda_0)^{-\alpha}$, one can derive the equation for T_d

$$T_d(r)^{4+\alpha} = \frac{\pi^4}{60 Q_0 \Gamma(4+\alpha)\zeta(4+\alpha)} \left(\frac{hc}{k\lambda_0}\right)^\alpha T_*^4 \left(\frac{R_*}{r}\right)^2$$

where Γ and ζ are the Gamma and Riemann zeta functions. Therefore, one finds that

$$T_d(r) \propto r^{-2/(4+\alpha)}$$

5. Variable Dust Temperature

In the previous example, we saw that T_d varies as a function of radius, so the emergent spectrum will be a composite of backbodies each radiating at temperature T_d. The observed flux will be

$$F_\nu = \frac{\int 4\pi j_\nu dV}{4\pi D^2}$$

$$F_\nu = \frac{\pi a^2 Q_\nu}{D^2} \int_{R_{\rm in}}^{R_{\rm out}} n_d(r) B_\nu(T_d(r)) 4\pi r^2 dr$$

where $R_{\rm in}$ and $R_{\rm out}$ are the inner and outer radii of the dust cloud.

If we use our previous result that $T_d = T_0(r_0/r)^{2/(4+\alpha)}$ and assume a power-law density distribution ($n_d \propto r^{-n}$), then one finds that

$$F_\nu \propto \nu^{\frac{n(4+\alpha)-(6+\alpha)}{2}}$$

6. Lyman α Heating

See Kwok Chapter 10.4.

Until now, we have only considered heating from direct starlight. However, in ionized environments, dust can also be heated by Lyα photons.

In the case of a cloud centrally heated by a star with significant ionizing radiation, the heating and cooling balance is given by

$$\pi \left(\frac{R_*}{r}\right)^2 \int_0^{\nu_1} (Q_\nu \pi a^2) B_\nu(T_*) d\nu + \frac{L_{\mathrm{Ly}\alpha}}{n_d V} = 4\pi \int_0^{\infty} (Q_\nu \pi a^2) B(T_d) d\nu$$

where $h\nu_1$ is the energy of a Lyman α photon.

7. Reflection and Scattering

See Kwok Chapter 10.5.

Until now, we have assumed that all incident starlight is absorbed by a dust grain. However, in many cases only a fraction (albedo = w) of the light is absorbed and the rest is scattered. Now we need to re-write the balance of heating and cooling, taking scattering into account. Again returning to the case of cloud internally heated by starlight, we find that

$$\frac{1}{4}\left(\frac{R_*}{r}\right)^2 (1-w)\left(\frac{\sigma T_*^4}{\pi}\right) = \int_0^\infty Q_\nu B_\nu(T_d) d\nu$$

In the IR, the dust is the dominant source of the observed emission. In the visible, reflected light will be the dominant source of observed flux. Therefore, one can measure the albedo of dust grains from the ratio of the observed visible and IR fluxes.

$$\frac{F_{vis}}{F_{IR}} = \frac{w}{1-w}$$

Given the values of T_d and w, one can then learn about the central heating source.

Dust grains also scatter NIR light, and deep NIR images of molecular clouds show both the direct emission from stars and protostars as well as the more diffuse scattered light from dust. Measurements of the scattered light at multiple NIR wavelengths can be used to map the dust density distribution.

8. Stochastic Heating

Until now, we have assumed that individual dust grains can be considered to be in thermal equilibrium due to equal rates of heating and cooling. This is actually only the case for big dust grains.

For very small grains, the energy of a single absorbed photon is significant compared to its internal heat energy, and the temperature of the grain will rise sharply and then cool due to self-radiation. Because $L \propto T_d^4$, these very small grains emit a lot while hot.

9. Interstellar Polarization

See Kwok Chapter 10.8

Light scattered by a dust grain causes the reflected light to be polarized. The resultant electric field vector will be perpendicular to the line between the dust grain and illuminating star.

If the dust is distributed with sperical symmetry around a star, the polarization pattern will be be circular and the integrated polarization will be zero. However, in the dust distribution has some other symmetry (or no symmetry), then the polarization pattern will show a corresponding symmetry.

Observations of starlight that has travelled through diffuse clouds often show that the light is slightly (a few %) polarized. The most likely interpretation is that dust grains in the ISM are not spherical and are not oriented randomly, and they are polarizing the starlight. The grains are thought to be aligned by magnetic fields. Therefore, observations of polarized starlight can be used to trace the direction and strength of the galactic magnetic field.

The FIR emission from dust grains can also be polarized because a grain will radiate most effectively when the electric field is parallel to the long axis of the grain. For instance, observations of the 1.3 mm emission from W51 show polarizations as large as 10%.