AY 102/126 PROBLEM SET 1

Ay102 students should solve problems 1,2,3. Due date **23 Jan 2007**
Ay126 students should solve all problems. Due date **22 Jan 2007**
Please hand in to Varun Bhalerao's mailbox by 5pm.

1. For an O6 star ($T_* = 3.8 \times 10^4\,\mathrm{K}$, $R_* = 5 \times 10^{11}\mathrm{cm}, M_* = 50\,\mathrm{M}_\odot$) embedded in a cloud of atomic hydrogen with density $n_\mathrm{H} = 10^2\,\mathrm{cm}^{-3}$ and temperature $T_0 = 25\,\mathrm{K}$:

   (a) Calculate the production rate of Lyman continuum photons assuming a blackbody spectrum.

   (b) Calculate the size of the Strömgren sphere (when it is first ionized).

   (c) The ionization fraction is defined as $y = n_p/n_H$. Equate the rates of ionization and recombination to calculate the ionization fraction at half this Strömgren radius.

   (d) The ionization timescale is the inverse of the rate of ionization. Similarly the recombination timescale is the inverse of the rate of recombination. Evaluate the timescales for ionization of a neutral hydgrogen atom and for recombination of a hydrogen ion at $0.1R_s$ and $0.9R_s$.

   (e) Estimate the thickness of the ionization front (the transition thickness from mostly HII to mostly HI).

   (f) Calculate the eventual size of the Strömgren sphere when it has reached pressure equilibrium with the ambient gas.

2. The intergalactic background at a redshift of $z = 3$ has an intensity at $h\nu_0 = 13.6$ eV of $J_\nu(\nu_0) = 10^{-21}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,Hz^{-1}\,ster^{-1}}$, produced by the light of distant quasars and young galaxies.

   (a) Estimate the Strömgren *length* of a young galaxy's outer disk, where $n_\mathrm{H} \simeq 0.1\,\mathrm{cm}^{-3}$.

   (b) What is the maximum total column density of hydrogen which can be kept ionized by the background light?

3. Stars are believed to form by gravitational collapse of dense interstellar clouds. Suppose that the infall of matter continues after the O6 star in problem (1) has turned on.

   (a) What is the accretion rate ($\mathrm{M_\odot\,yr^{-1}}$) required for the HII region to be entirely suppressed?

   (b) What accretion rate will limit the size of the HII region to 0.15 pc? You may assume that the accreting gas follows a free-fall velocity law.

4. A stellar wind with density distribution $n(r) = n_0(r/r_0)^{-2}$ is being ionized by a central star with a total Lyman continuum photon output of Q.

   (a) Calculate the number of ionizing photons remaining as a function of radius.

   (b) Find the minimum value of Q for which the entire stellar wind is ionized.

   (c) For Q smaller than this critical value, show that the Strömgren radius is given by,

   $$r_s = \frac{r_0}{1 - Q/Q_1} \tag{1}$$

   where $Q_1 = 4\pi r_0^3 n_0^2 \alpha_B$.

   (d) For a star of temperature $50000\,K$ and a luminosity of $50000\,L_\odot$, calculate the maximum mass loss rate of the stellar winf that can be fully ionized by the star. Assume the stellar wind begins at the stellar surfacec, has a gas temperature of $10000\,K$, and is expanding at a speed of $2000\,kms^{-1}$. *Kwok, problem 4.3*