AY 102/126 PROBLEM SET 3

Ay102 students should solve problems 1, 2(a), 2(b) and 3. Due date **20 Feb 2008**
Ay126 students should solve all problems. Due date **19 Feb 2008**
Please hand in to Laura Pérez's mailbox by 5pm.

1. Derive the net heat input into a gas cloud due to the formation of molecular hydrogen on dust grains. The binding energy of hydrogen molecules to the grain surface is 0.025 eV. Assume:
   (a) the formation rate of molecular hydrogen is 10^{-17} cm^{3}s^{-1} $\times\, n_{\rm HI} \times n_{\rm H_2}$.
   (b) $n_{\rm HI} = n_{\rm H_2} = 100$ cm^{-3} and kinetic temperature of 100 K.
   (c) equipartition of energy between the translational, rotational, and vibrational modes, where energy from the latter two is radiated away.

2. A dust grain with radius a is near a star with luminosity L_*. Assume that the grain emissivity is given by $Q_{abs}(\lambda) = Q_0(\lambda_0/\lambda)^n$, where λ is wavelength and n is the power-law index for the wavelength dependence of the grain emissivity.
   (a) Calculate the dust temperature as a function of n, $T_d(r, n)$, where r is distance from the star. You can assume that the star radiates as a blackbody, and the dust grains radiate as a modified blackbody.
   (b) Now, you will calculate the Spectral Energy Dustribution (SED) of a star plus a disk, assuming that the central star is just like our Sun, and that the inner and outer cutoff radii of the disk are given by:
      i. R_{inner} = condensation boundary of silicates (Assume that the grains will sublimate at a temperature of 1800 K).
      ii. $R_{outer} \sim 2 \times 10^2$ AU, which is comparable to the size of disks seen in silhouette against the Orion Nebulae (for pretty pictures visit: http://www.aip.de/~mjm/Astronomy/ImageGallery.html, under "Circumstellar disks", or read McCaughrean & O'Dell 1996)

      To compute the observed SED (that means to plot λF_λ [cgs units] vs λ [μm] on log-log scale) you can assume:
      i. The disk is optically thick (very important! otherwise you will have to deal with the density profile within the disk (radial and vertical), the grain size distribution, etc.).
      ii. The dust grains and the central star radiate as a perfect blackbody (n=0).
      iii. We are viewing the disk face-on.
      iv. The distance to the system is 10 pc.

      We want you to use two different disk temperature profiles: the one you calculated in (a) for $n = 0$, and the Chiang & Goldreich model for a thin flat disk, given by:

$$T_d(r) = T_* \left(\frac{2}{3\pi}\right)^{1/4} \left(\frac{R_*}{r}\right)^{3/4}$$

      where the main difference with model (a) is that they actually take into account geometry. Luckily enough, we don't ask you to derive this, but you can always read the paper.
      You have to produce two plots (one for each model), with the SED from the star, the disk, and the star+disk system (i.e. three different curves). Also hand in the code you wrote.
   (c) **Grad. students ONLY:** As the disk evolves it could form a planet, and this planet will clear out the inner region of the disk. Assuming that the disk was cleared out up to the orbital distance of Jupiter, but then continues normally, plot the observed emission from the system star + disk. What do you notice? What wavelengths are more suitable to study such systems?

3. Cloud Collapse and Fragmentation:

   (a) Estimate the length scale at which the gravitational forces balance the thermal pressure, for an isolated cloud of gas with temperature T, density ρ and molecular weight μ (this is a crude estimate of the Jeans length). Convert this to a mass scale, which is called the Jeans mass.

   (b) What happens to a gas cloud smaller than this length scale? how about larger?

   (c) Giant Molecular Clouds (GMC) have typical temperatures of $T \sim 15$ K, densities of $n \sim 100$ cm^{-3} and masses ranging from $10^5 - 10^6 M_\odot$. For this temperature and density, what is the mass of the smallest gas cloud for which gravity overwhelms internal pressure?

   (d) Does the limit calculated in (c) mean that stars will form only with high mass?, what would be the physical process that leads to formation of less massive stars?. Why does a GMC with a total mass larger than this limit not collapse completely?

   (e) If the collapse of a cloud is isothermal the Jeans mass will decrease as the density increases, hence making it easier for different regions of the cloud to collapse independently and fragment. But it appears that there is no limit for how small a fragment will be, since as you get more and more dense you can fragment into smaller and smaller objects. The goal of this part is try to estimate the minimum mass for which fragmentation will be stopped.

      i. For each of these types of collapse, how does the Jeans mass scale with density?

      ii. Calculate what is the energy released by contraction and the timescale for free-fall.

      iii. If the gravitational energy that is released during collapse can be radiated away efficiently (i.e. $L_{cloud} = L_{grav} \sim \Delta E_{grav}/t_{ff}$) you can keep the temperature nearly constant and have an isothermal collapse. But if you can't transport efficiently the energy out of the cloud (i.e. $L_{cloud} = eL_{rad}$, where $e < 1$ is an efficiency factor), the temperature will rise and the collapse will be adiabatic. The minimum mass of a fragment arises from the transition between isothermal and adiabatic collapse.
      Assuming that you are just in the transition point from isothermal to adiabatic, and further assume that the cloud is optically thick and in thermodynamic equilibrium, calculate the minimum Jean mass for a fragment. Evaluate for typical values: $T \sim 1000K$, $e \sim 0.1$.

4. Suppose you make CO (1-0) measurements of a molecular cloud at $l = 30°$, $b = 0°$ and doppler velocity relative to the earth of 60 km/s. The gas temperature in the cloud is 20 K.

   (a) Assuming the sun is at galactic radius 8.5 kpc and that the galaxy rotates at constant circular velocity of 220 km/s everywhere outside a radius of 1 kpc, calculate the galactic radius and distance from the sun of the cloud.

   (b) Suppose the angular diameter of the cloud is 10 arcmin and the width of the CO line is 5 km/s, calculate the clouds virial mass (state any assumptions).

   (c) Suppose you deduce from looking at the ^{13}CO and ^{12}CO lines that the ^{12}CO line has an optical depth of 50. Estimate the critical density of the CO(1-0) transition.

   (d) Given the mass you estimated in (b), estimate the mean density, compare with (c) and evaluate what the excitation temperature of the CO is in terms of the gas temperature.

   (e) How is the brightness temperature of the CO emission line related to the cosmic background temperature (2.75 K) and the gas temperature?