Ay 102/126 PROBLEM SET 4

Ay102 students should solve problems 1 and 2. Due date **March 5, 2008**
Ay126 students should solve all problems. Due date **March 4, 2008**
Please hand in to Laura Pérez's mailbox by 5pm.

1. A shock is moving at 100 km s^{-1} into neutral gas with Hydrogen number density 10 cm^{-3} and temperature $T = 10^4$ K. Assume a monatomic gas, negligible magnetic fields and molecular weight for the neutral gas $\mu = 1.4$.
   (a) What is the sound speed in the unshocked gas and the Mach number of the shock?
   (b) Ignoring energy lost to ionization, what will be the immediate temperature of the post-shock gas? What is the compression factor across the shock?.
   (c) What would be the postshock temperature if the gas entering the shock were ionized? Assume $\mu = 0.70$ for the fully ionized gas.
   (d) If the preshock gas is threaded by a purely transverse magnetic field of 30 μG, what is the sound speed, the Alfven speed and the magnetosonic speed in the gas? What is the maximum compression factor across the shock in this case?

2. A shock wave produced by a supernova explosion is moving into neutral atomic gas at 250 km s^{-1}.
   (a) Rewrite the jump condition for energy flux to include the effect of ionization of hydrogen (denote the ionization potential by χ).
   (b) Estimate the temperature immediately behind the shock, both (i) with, and (ii) without taking into account the ionization energy.
   (c) For case (i), estimate the thickness of the layer in which the gas behind the shock cools from 10^6 K to 10^4 K, assuming that the number density of the unshocked gas is 1 cm^{-3} and that the cooling rate can be described by a single value (cf. the figure in problem 3, homework 2).
   (d) Also for case (i), estimate the time that the shock has taken to slow down from an initial speed of 10^4 km s^{-1} if the initial energy input was 10^{51} erg.
   (e) Again for case (i): what is the maximum density of post-shock gas which is attained in this supernova remnant?

3. Use the conservation of momentum equation with magnetic field terms included:

$$\rho\left(\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}$$

together with the continuity equation and the Maxwell equation:

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0, \quad \frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) + \frac{\eta}{4\pi}\nabla^2\vec{B}$$

to show that disturbances transverse to the magnetic field $\vec{B}$ will travel along $\vec{B}$ at the Alfvén speed. (Hints: You can assume that the resistivity is negligible and that $P\rho^{-\gamma}$ = constant. Linearize each one of these equations and look for solutions of the form: $\rho = \delta\rho\, e^{i(wt-\vec{k}\cdot\vec{x})}$, for density, pressure, velocity and B perturbations. You'll find a dispersion relation that can then be analyzed for the specific case of disturbances that travel perpendicular to the magnetic field)