AY 102/126 PROBLEM SET 5

Ay102 students should solve problems 1,2,4. Due date **12 Mar 2008**
Ay126 students should solve all problems. Due date **12 Mar 2008**
Please hand in to Varun Bhalerao's mailbox by 5pm.

1. Suppose that a supernova blast wave is expanding into a cavity which was evacuated by the progenitor star's stellar wind. Assume that the kinetic energy of the blast wave as a function of time can be parametrized by

   $$E(t) = E_0 t^{\alpha}$$

   and that the pre-shock density in the cavity is described by the power law

   $$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\beta}$$

   for scaling constants r_0 and ρ_0.

   (a) Derive an expression for the radius of the blast wave $r(t)$ [a more general version of the Taylor-Sedov solution derived in class].

   (b) For the case $\alpha = 0$, derive the value of β which separates decelerating (Rayleigh-Taylor stable) from accelerating (Rayleigh-Taylor unstable) blast waves.

2. Cosmic Ray Ionization and Ambipolar Diffusion:

   One of the sequences of reactions that leads to the ionization of H_2 molecules is the following:

   (1) $H_2 + p \rightarrow H_2^+ + e^- + p$, in which a high-energy cosmic ray proton ejects an electron from the H_2 molecule. The rate for this can be expressed in the form $Jn(H_2)$, where $J \simeq 10^{-17}$ s^{-1} if the density is in cgs units.
   (2) $H_2^+ + H_2 \rightarrow H_3^+ + H$; the rate for this can be expressed in the form $\alpha_2 n(H_2) n(H_2^+)$, where α_2 is a suitable recombination coefficient.
   (3) $H_3^+ + e^- \rightarrow 3H$. The rate for this can be expressed in the form $\alpha_3 n(H_3^+) n_e$.

   (a) Show that in steady state this will give $n_e/n(H_2) \propto [J/\alpha_3 n(H_2)]^{1/2}$

   (b) Also show that the ratio between the ambipolar diffusion time scale and the free-fall time scale of a molecular cloud is

   $$\frac{t_{ad}}{t_{ff}} \approx 3 \left(\frac{n_{\mathrm{H}_2}}{n_{\mathrm{H}}}\right) \sqrt{\frac{J}{10^{-17}\,\mathrm{s}^{-1}}}$$

   Use $\alpha_3 = 4 \times 10^{-6}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$

   Hint: H_2^+ is a transient species, you can neglect its contribution to the net positive charge in the cloud.

3. Angular Momentum Loss Through Alfven Waves

Consider a spherically symmetric cloud of radius R rotating with an angular velocity ω in a region containing an axially symmetric magnetic field B that is parallel to the axis of rotation. Surrounding this spherical cloud is the ISM with ρ=4×10^{-23} g cm^{-3} and T=50 K. You may assume (1) conductivity is very high so that flux freezing occurs in the medium, (2) the ISM plasma surrounding the rotating cloud is initially at rest and later on picks up only a v_θ component of velocity. (3) The magnetic field has only θ and z components at all times.

(a) As the Alfven waves propagate through the plasma, they set the plasma in rotation thereby transferring angular momentum from the cloud to the plasma. Derive an equation for the rate of loss of the cloud's angular momentum.

You may also assume that other physical effects stop this corotation at some distance $\simeq$R and that the velocity structure is of the form $v(t-(z/v_A),r) = \omega(t)r$ (where v_A is the Alfven velocity).

(b) A condensation with M=10M$_\odot$ has an average density of $\rho \simeq 4 \times 10^{-17}$ g cm^{-3} when the fragmentation stops. At this time its angular velocity is approximately $\omega(0)$=2×10^{-15} s^{-1} and B field is 3 microgauss.

What is the typical angular velocity of a main-sequence star? Assuming angular momentum is conserved in the final stage, what angular velocity does this correspond to at the above cloud radius? How long will the cloud take to decrease its angular velocity to this value?

How does it compare to the free-fall time assuming an initial radius of 0.1 pc?

Hint: It might be helpful to look at Chapter 23 in Bowers and Deeming.

4. A fairly realistic model of the ISM in the disk consists of gas in three phases:

(i) a 1 kpc thick layer of ionized coronal gas with $n_{\rm H} \simeq 3 \times 10^{-3}\,{\rm cm}^{-3}$ and $T \simeq 10^6\,{\rm K}$, in which are embedded

(ii) clouds of the WIM with $n_{\rm H} \simeq 0.3\,{\rm cm}^{-3}$, $T \simeq 8000\,{\rm K}$, radius $\simeq 5\,{\rm pc}$, and number density $n_{\rm WIMC} \simeq 3 \times 10^{-4}\,{\rm pc}^{-3}$, and

(iii) clouds of the CNM with $n_{\rm H} \simeq 30\,{\rm cm}^{-3}$, $T \simeq 100\,{\rm K}$, radius $\simeq 2.5\,{\rm pc}$, and number density $n_{\rm CNMC} \simeq 3 \times 10^{-4}\,{\rm pc}^{-3}$ ($n_{\rm WIMC} \simeq n_{\rm CNMC}$ because the CNM clouds are often the dense cores of WIM clouds).

Thus, spherical clounds of CNM are assumed to be embedded inside shells of WNM, which are present in coronal gas. CNM, WIM and the coronal gas are mutually exclusive - take the proper geometry into consideration !

(a) Along the line of sight to a star 1 kpc away in the disk, estimate the number of WIM clouds and the number of CNM clouds intercepted.

(b) Suppose the star in question is a pulsar. Compute the contributions to the dispersion measure from each of the three phases of the ISM in units of ${\rm cm}^{-3}\,{\rm pc}$.

(c) Now look along a line of sight leaving the Galaxy in the vertical direction. Compute the contributions to the emission measure from each of the three phases of the ISM in units of ${\rm cm}^{-6}\,{\rm pc}$.

(d) Calculate and compare the mean contributions to the mean gas density (averaged over a very large volume, e.g. $10^6\,{\rm pc}^3$) from each of the three phases of the ISM. What fraction of the volume of the ISM is occupied by each phase?