Ay 20 Basic Astronomy and the Galaxy
Problem Set 2

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1 Sun as an (approximate) black body

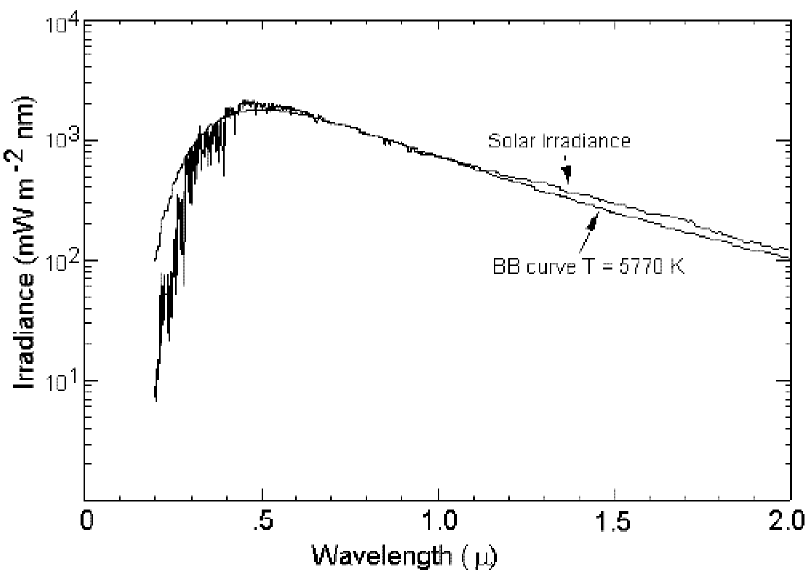
a) Integrate We have to calculate the integrated intensity of a blackbody with a temperature $T_\odot$=5777 K. *(Note that the symbol $\odot$ always refers to the sun, $\oplus$ refers to earth.)* The specific intensity of a blackbody is given by,

$$B_\lambda = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda k T_\odot}} - 1}. \tag{1}$$

This integral is solvable analytically, and was outlined in class. For this problem set, it is sufficient to put the numbers in your favourite math package and get the numerical answer. A perl code for evaluating this integral is available at http://www.astro.caltech.edu/~varun/ay20/ps2/. The correct answer is,

$$\int_0^\infty B_\lambda(T_\odot) d\lambda = 2 \times 10^{10}\,\mathrm{erg\,cm^{-2}s^{-1}sr^{-1}} = 2.01 \times 10^7\,\mathrm{J\,m^{-2}s^{-1}sr^{-1}}. \tag{2}$$

Just as a matter of interest, the figure below shows the actual solar irradiance compared to that of a black body at the effective temperature of the sun (i.e. the black body with the same total energy output as the sun). Looks pretty close, but beware of the log scale!



b) Stephan–Boltzmann radiation constant The Stephan–Boltzmann radiation constant is $\sigma = 5.67 \times 10^{-5}\,\mathrm{erg\,cm^{-2}\,s^{-1}} = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}}$. Putting in the numbers we get,

$$\frac{\sigma T^4}{\pi} = 2.01 \times 10^{10}\,\mathrm{erg\,cm^{-2} s^{-1} sr^{-1}} = 2 \times 10^{7}\,\mathrm{J\,m^{-2} s^{-1} sr^{-1}} \tag{3}$$

which is numerically equal to the value we obtained in Equation 2.

c) Photodiode band The `S2386-18K` photodiodes are sensitive in the wavelength range 300 nm$<\lambda<$1100 nm. The flux emitted in this range by a blackbody at $T_\odot$ is,

$$\int_{300\,nm}^{1100\,nm} B_\lambda(T_\odot)d\lambda = 1.46 \times 10^{10}\,\mathrm{erg\,cm^{-2} s^{-1} sr^{-1}} = 1.46 \times 10^{7}\,\mathrm{J\,m^{-2} s^{-1} sr^{-1}}. \tag{4}$$

Thus, 73% of the sun's energy comes out in the range of wavelengths to which the photodiode is sensitive.

2 Flux from the sun in a clever way

a) Solid angle of the sun There seems to be a lot of confusion over the formula given in the texbook for solid angle. Recall that an angle is the ratio of the length of an arc to the radius. Similarly, solid angle is the ratio of area on a sphere's surface the square of the sphere's radius. Let us denote the radius of the sun by $R_\odot$ and the earth–sun distance by d. So the solid angle is given by,

$$\Omega_\odot = \frac{\text{Projected area of sun}}{(\text{Earth sun distance})^2} = \frac{\pi R_\odot^2}{d^2}. \tag{5}$$

But, we know that $\theta = \frac{2R_\odot}{d}$ is the angular diameter of the sun, 32' (0.0093 radians). So, we get

$$\Omega_\odot = \pi \cdot \left(\frac{\theta}{2}\right)^2 = 6.8 \times 10^{-5}\,\text{steradians} \tag{6}$$

b) Flux from solid angle In part 1(a) we showed that the energy emitted by the sun per unit area per unit time per unit solid angle is given by I $= \sigma T^4/\pi$. We can remove the dependence on solid angle by integrating along all the outward directions:

$$I^* = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I cos(\theta) \cdot sin(\theta) d\theta = \pi I \tag{7}$$

This is nothing but the Stephan–Boltzmann radiation law, $I^* = \sigma T^4$. The total luminosity of the sun is then given by the product of I^* and surface area. The flux at earth ($F_\oplus$) is obtained by spreading this total power over an area of $4\pi d^2$ where d is the earth–sun distance.

$$F_\oplus = \frac{L_\odot}{4\pi d^2} = \frac{I^* \cdot 4\pi R_\odot^2}{4\pi d^2} \tag{8}$$

Combining with Equation 5 and substituting $I^* = \pi I$ we get $F_\oplus = I(T_\odot)\Omega_\odot$. Numerically, we get $F_\oplus = 1360\,\mathrm{W\,m^{-2}} = 1.36 \times 10^6\,\mathrm{erg\,cm^{-2}\,s^{-1}}$.

c) Flux from luminosity For this calculation we use the radius of the sun $R_\odot = 6.96 \times 10^8$ m, earth–sun distance $a = 1.5 \times 10^{11}$ m, $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}}$ and $T_\odot = 5777$K. Thus, we get $L_\odot = 3.84 \times 10^{26}$ W. The flux at earth is then F $= L_\odot/(4\pi a^2) = 1360\,\mathrm{W\,m^{-2}}$, same as in Part (b).

3 Photon counts from a blackbody

a) Photons in photodiode passband The energy emitted by a blackbody in a wavelength range dλ centered on the wavelength λ is B$_\lambda$dλ. Hence the number of photons in this range is obtained by dividing this value by the energy of a photon, hc/λ. Put the integral in your favourite numerical package or your own code, to get the answer:

$$\int_{300\,nm}^{1100\,nm} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 4.8 \times 10^{21}\,photons\,cm^{-2}\,s^{-1}\,sr^{-1} \tag{9}$$

A perl code for evaluating this integral is available at http://www.astro.caltech.edu/~varun/ay20/ps2/.

b) Photons in V–B–U bands Since we are assuming 100% transmission in the bands and 0% outside, this is simply the same integral as 9:

$$n_V = \int_{505\,nm}^{595\,nm} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 6.4 \times 10^{20}\,photons\,cm^{-2}\,s^{-1}\,sr^{-1} \tag{10}$$

$$n_B = \int_{391\,nm}^{489\,nm} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 5.4 \times 10^{20}\,photons\,cm^{-2}\,s^{-1}\,sr^{-1} \tag{11}$$

$$n_U = \int_{331\,nm}^{399\,nm} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 2.5 \times 10^{20}\,photons\,cm^{-2}\,s^{-1}\,sr^{-1} \tag{12}$$

The same perl code above (http://www.astro.caltech.edu/~varun/ay20/ps2/) solves these integrals too.

4 Photon counts from stars

a) Photon flux First let us calculate the solid angle of the sun if it was at 10 pc. 10 pc $= 2 \times 10^6$ AU. Since $\Omega \propto r^{-2}$, we get:

$$\Omega_\odot(10\,pc) = \Omega_\odot(1\,AU) \times \frac{(1\,AU)^2}{(10\,pc)^2} = \frac{6.8 \times 10^{-5}}{(2 \times 10^6)^2} = 1.7 \times 10^{-17}\,\text{steradians} \tag{13}$$

The total number of photons emitted by the sun per unit area per unit time per unit solid angle is given by,

$$n_{tot} = \int_0^\infty \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 9.3 \times 10^{21}\,photons\,cm^{-2}\,s^{-1}\,sr^{-1} \tag{14}$$

Hence the number flux of photons at earth from such a star (with M$_V$ = 4.79, d = 10 pc) is,

$$n_* = n_{tot} \times \Omega_\odot(10\,pc) = 1.6 \times 10^5\,photons\,cm^{-2}s^{-1} \tag{15}$$

b) V–band photons seen by the eye Here we need to assume a diameter for the pupil of the eye. For a dark–adapted eye, the pupil dialates and its diameter is about 6 mm. Thus the area of the pupil is about $0.28\,\mathrm{cm}^2$. Proceeding as in Part (a), we get:

$$n_{eye} = n_V \times \Omega_\odot(10\,pc) \times Area\ of\ eye \times \Delta t = 100\,\text{photons} \tag{16}$$

c) V–band photons at Palomar Proceeding like Part (a) we calculate that we receive 1.1×10^4 photons $\mathrm{cm^{-2}\,s^{-1}}$ from a star with m_v=4.79. If we used the P–60 to look at this star for 5 minutes, the number of photons collected would be,

$$n_{P60} = \pi 75^2\,\mathrm{cm}^2 \cdot 300\,\mathrm{s} \cdot n_v \cdot \Omega_\odot \tag{17}$$

$$n_{P60} = \pi 75^2\,\mathrm{cm}^2 \cdot 300\,\mathrm{s} \cdot 1.1 \times 10^4\,\text{photons}\,\mathrm{cm^{-2}\,s^{-1}} = 5.7 \times 10^{10}\,\text{photons} \tag{18}$$

To get 1000 photons in an exposure, the target star must be $1000/8.5\times10^{10}$ fainter than this source. Since we know the ratio of their fluxes[1], we can use our standard formula to find the apparent magnitude of the target star:

$$m_1 - m_2 = -2.5\log\left(\frac{F_1}{F_2}\right) \tag{19}$$

$$m_{V,target} - 4.79 = -2.5\log\left(\frac{1000}{5.7\times10^{10}}\right) \tag{20}$$

This gives $m_{V,target}$ = 24.2 as the limiting magnitude for P–60. In reality the limit is lower (brighter) than this due to sky background etc.

d) U, B–band photons at Palomar We have to repeat the complete procedure of Part (c) here. But there is another method which I find shorter. Let us take a look at Equation 20 again, but without substituting the numbers

$$m_{target} - M_{target} = -2.5\log\left(\frac{1000\,\text{photons}\,\mathrm{cm^{-2}\,s^{-1}}}{\text{Area of telescope}\cdot\text{Integration time}\cdot n_{photons}}\right) \tag{21}$$

If we write this equation for two bands and subtract, we get:

$$m_U - m_V = M_U - M_V + 2.5\log\left(\frac{n_U}{n_V}\right) \tag{22}$$

For the U–band we have M_U = 5.51, n_U=2.5×10^{20} photons $\mathrm{cm^{-2}\,s^{-1}}$, giving m_U=23.9. For the B–band, M_B=5.41, n_B=5.4×10^{20} photons $\mathrm{cm^{-2}\,s^{-1}}$, so m_B=24.6.

e) Distance to the star For calculating the distance we use the distance modulus:

$$m - M = 5\log(d) - 5 \tag{23}$$

$$d = 10^{\frac{m_V - M_V + 5}{5}} = 76.2\,\mathrm{kpc} \tag{24}$$

[1]Note that this is really a ratio of number of photons. It will equal ratio of fluxes only if all the photons have roughly the same energy