

Ay 20 Basic Astronomy and the Galaxy

Problem Set 4

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1 The biggest stellar mass black hole

The problem is based on a Nature paper: Orosz et al., “A 15.65-solar-mass black hole in an eclipsing binary in the nearby spiral galaxy M 33”, 18 Oct 2007, Nature, 449, 872. You can access the paper online at <http://www.nature.com/nature/journal/v449/n7164/full/nature06218.html>.

a) Temperature of the companion star Let us begin with Figure 8.11 in Carroll & Ostlie. The star spectrum shows a Si IV line at $\sim 4090 \text{ \AA}$. From the figure we see that the Si IV line is present only in stars hotter than B0 (25000 K). The figure also shows that the He II line is visible only above about 35000 K. The companion spectrum shows one He II line at 4540 \AA but the feature at $\sim 4680 \text{ \AA}$ is missing. Thus we choose our temperature as the point at which the Si IV line has maximum strength, and He II has started appearing - this corresponds to a companion temperature $T_* \sim 35000 \text{ K}$.

Orosz et al. determine the temperature to be 35000 K using detailed spectral fits. The spectral class of the companion star is O7III or O8III.

b) Stellar radius Note that we should be careful and use only the V-band fluxes and luminosities in these calculations. The temperature of the companion is 35000 K, hence the radiation peaks at 85 nm, and most of the radiation will be in ultraviolet. Using bolometric luminosities in calculations will give incorrect answers.

There are multiple similar ways of solving this. Here, we will calculate the radius using sun as the reference star. We will use the subscript \odot for the sun and $*$ for the companion star. First evaluate I_V for the sun and the companion star, then calculate the flux of the star using the apparent magnitudes. This will give us the solid angle of the star Ω_* , which coupled with the distance gives us its radius.

Let us begin by calculating $I_{V\odot}$ and I_{V*} . You have used these integrals several times in previous problem sets:

$$I_{V\odot} = \int_{505 \text{ nm}}^{595 \text{ nm}} B_\lambda(5777 \text{ K}) d\lambda = 2.3 \times 10^9 \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (1)$$

$$I_{V*} = \int_{505 \text{ nm}}^{595 \text{ nm}} B_{\lambda}(35000 \text{ K}) d\lambda = 2.0 \times 10^{11} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \quad (2)$$

Next, we use the solid angle of the sun, $\Omega_{\odot} = 6.8 \times 10^{-5} \text{ sr}$ (Problem set 2, Equation 6) to find the flux from the sun:

$$F_{V_{\odot}} = I_{V_{\odot}} \Omega_{\odot} = 1.6 \times 10^5 \text{ ergs cm}^{-2} \text{ s}^{-1} \quad (3)$$

The apparent visual magnitude of the sun is given on the front cover of Carroll & Ostlie: $m_{V_{\odot}} = -26.8$. The apparent visual magnitude of the companion star is $m_{V*} = 18.4$. Thus we can calculate the flux from the star,

$$F_{V*} = F_{V_{\odot}} \cdot 10^{(18.4+26.8)/-2.5} = 1.3 \times 10^{-13} \text{ ergs cm}^{-2} \text{ s}^{-1} \quad (4)$$

$$\Omega_* = \frac{F_{V*}}{I_*} = 6.7 \times 10^{-25} \text{ sr} \quad (5)$$

Using the distance to the star, $d_* = 840 \text{ kpc}$, we get the radius of the star,

$$R_* = d_* \sqrt{\Omega_*/\pi} = 1.2 \times 10^{12} \text{ cm} \quad (6)$$

To put things in perspective, we note that $R_* = 17.2 R_{\odot} = 0.08 \text{ AU}$. This justifies the spectral class III which we had just stated in part a: the star is O7III or O8III - III stands for giant. Orosz et al. calculate R_* to be $19.6 R_{\odot}$.

c) Stellar luminosity Now we have to calculate the bolometric luminosity of the star. Since we are assuming the star radiates as a blackbody, we have:

$$\frac{L_*}{L_{\odot}} = \frac{4\pi R_*^2 \sigma T_*^4}{4\pi R_{\odot}^2 \sigma T_{\odot}^4} \quad (7)$$

$$L_* = 17.2^2 \times \left(\frac{35000}{5777} \right)^2 L_{\odot} = 4 \times 10^5 L_{\odot} \quad (8)$$

d) Semi major axis of orbit

i. Semi major axis from eclipse duration Measure the eclipse duration from the figure. Let us denote the eclipse duration by t_e and the binary period by P . We get $t_e = 0.45 \text{ d} = 3.9 \times 10^4 \text{ s}$. If $\sin i = 1$, the eclipse duration corresponds to the companion star moving by $2R_*$ relative to the black hole. This gives the velocity of the primary:

$$v_* = \frac{2R_*}{t_e} = 6.2 \times 10^7 \text{ cm s}^{-1} = 620 \text{ km s}^{-1} \quad (9)$$

This velocity is also related to the semimajor axis of the orbit:

$$v_* = \frac{2\pi a}{P} \quad (10)$$

$$a = 2.9 \times 10^{12} \text{ cm} = 2.44 R_* = 0.196 \text{ AU} \quad (11)$$

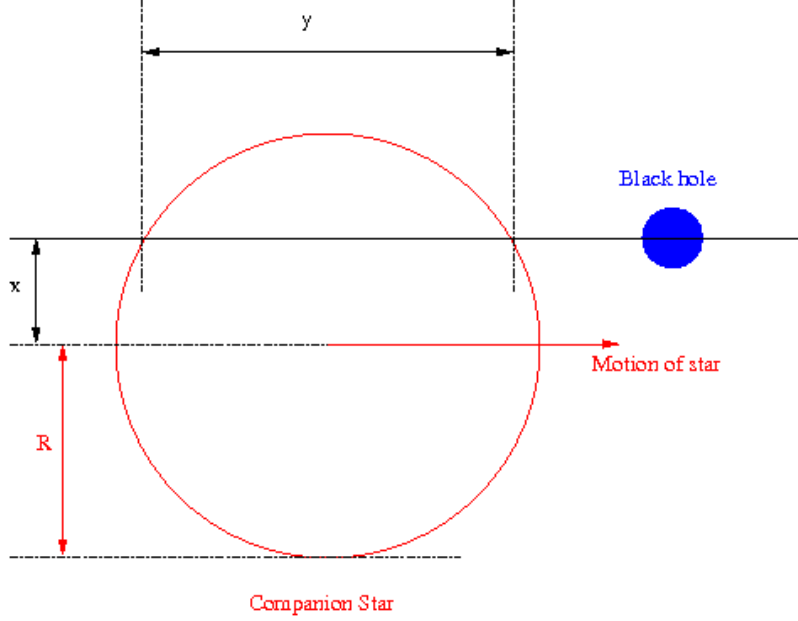


Figure 1: Geometry of non-central eclipse ($\sin i < 1$)

ii. Changing $\sin i$ The case $\sin i < 1$ corresponds to a case like Figure 1. Using symbols from the figure, we have $x = a \cdot \cos i$, $y = R_* \sqrt{1 - x^2}$. Thus, the star moves a distance y in the eclipse. But since we have assumed $\sin i = 1$, we still proceed with calculations as in Part i. Thus, we end up calculating a *higher* velocity for the star by assuming $\sin i = 1$ instead of using a true value of $\sin i < 1$. This in turn will lead to a *larger* value for the inferred a as compared to the true a (which would have been obtained by using the correct value of $\sin i$).

e) Lower limit for combined mass Kepler's third law is,

$$\frac{a^3}{P^2} = \frac{G(M_1 + M_2)}{4\pi^2} \quad (12)$$

If we measure mass in solar masses, period in years and semi-major axis in AU, we can rewrite Equation 12 as:

$$\frac{a^3(\text{AU})}{P^2(\text{yr})} = \frac{(M_1 + M_2)}{M_\odot} \quad (13)$$

This gives $M_1 + M_2 = 83.5M_\odot$. Note that in Part d ii, we inferred that the value of a we have is the lower limit of possible values of a . Hence, the value calculated in Equation 13 is a lower limit on the total mass of the system.

f) Mass function The mass function is given in Equation 7.7 of Carroll & Ostlie:

$$\frac{M_2^3}{(M_1 + M_2)^2} \sin^3 i = \frac{P}{2\pi G} v_{1r}^3 \quad (14)$$

In our case, M_2 is M_b , mass of the black hole. From the radial velocity curve we get $v_{1r} = 110 \text{ km s}^{-1}$. Thus, the mass function is:

$$\frac{M_b^3}{(M_* + M_b)^2} \sin^3 i = 9.5 \times 10^{32} \text{ g} = 0.48M_\odot \quad (15)$$

g) Individual masses We substitute $M_b + M_*$ from Equation 13 and assume $\sin i=1$ to get $M_b = 14.9M_\odot$ and $M_* = 68.6M_\odot$. These are in good agreement with the value obtained by Orosz et al., who get $M_b = 15.7M_\odot$ and $M_* = 70M_\odot$.

h) Position on the mass–luminosity relation The companion is off the upper right side of Figure 7.7 of Carroll & Ostlie. This is expected as the figure only contains stars cooler than and up to class B.

i) $\sin i < 1$ For this discussion we refer to Figure 1 again. Decreasing $\sin i$ to 0 corresponds to changing the inclination of the orbit from edge–on to face–on. The lowest value of $\sin i$ where we can still just see the eclipse, corresponds to the configuration where $x = R_*$. Since $x = a \cdot \cos i$, this gives a limit that $\sin i > 0.91$ for seeing an eclipse.

Let us consider the cases qualitatively. If $\sin i < 1$ then we have to use y instead of $2R_*$ in Equation 9. This decreases the calculated value of a in Equation 10. Note that this is opposite of the discussion in Part d ii, where we were considering the (erroneous) assumption that $\sin i = 1$ instead of the true lower value. A lower value of a leads to a lower total mass $M_b + M_*$ in Equation 13. The mass function (Equation 15) gives lower values for the individual masses, too. This is shown qualitatively in Figure 2.

The ratio of masses of the individual stars is a bit more complicated. A lower value of $\sin i$ means $y < 2R_*$, and in turn lower values of v_* relative to the black hole. Let u_* and u_b denote the velocities of the star and black hole respectively in the center of mass frame. We have a fixed

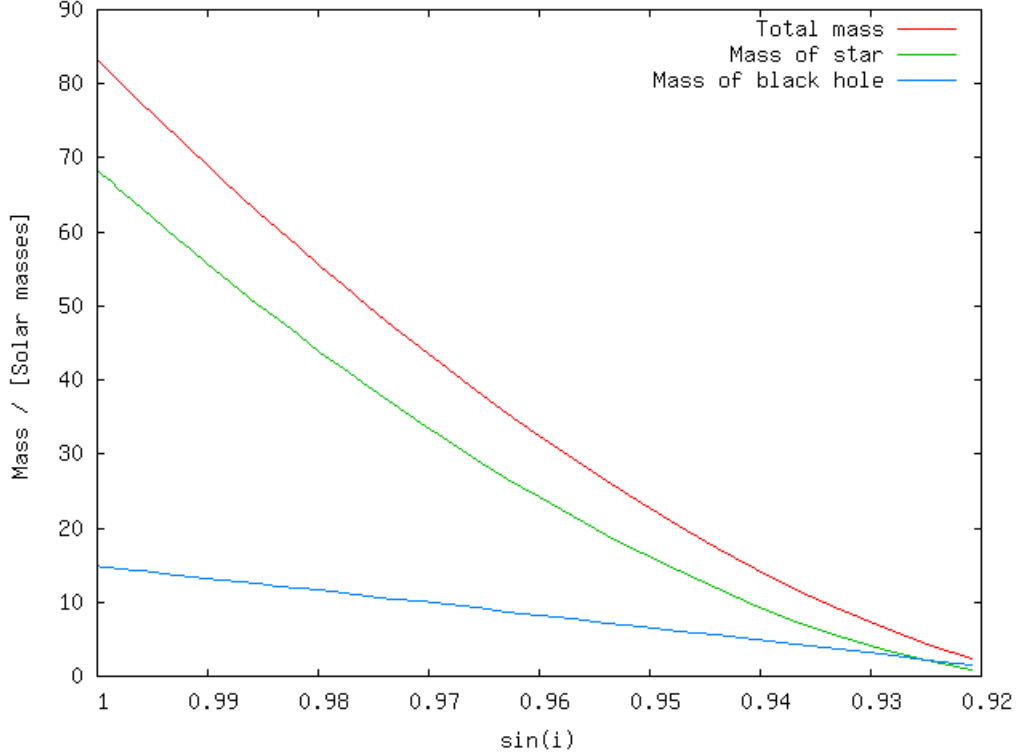


Figure 2: Variation of total mass and masses of individual components with $\sin i$

value of the velocity amplitude of the companion star in the center of mass frame, $u_* = 110 \text{ km s}^{-1}$. Since v_* is the relative velocity of two bodies moving in opposite directions, we have:

$$v_* = u_* + u_b \quad (16)$$

Also, for binary systems,

$$\frac{u_b}{u_*} = \frac{M_*}{M_b} \quad (17)$$

$$\frac{v_*}{u_*} = 1 + \frac{M_*}{M_b} \quad (18)$$

Since u_* is fixed, a lower value of v_* results in a lower value of the ratio M_*/M_b . This can be seen in Figure 3.

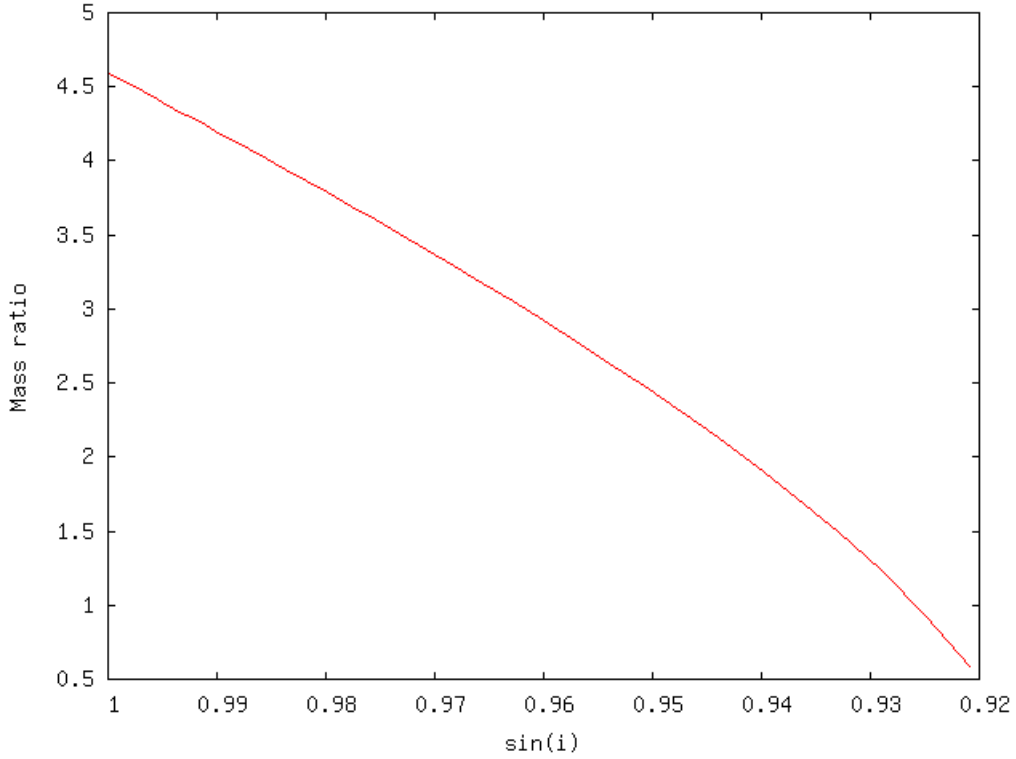


Figure 3: Variation of ratio of companion star mass M_* to black hole mass M_b with $\sin i$

2 Barnard's star

a) **Radial velocity** We observe the H_α in air at a wavelength of $\lambda_0 = 656.281 \text{ nm}$. The radiation from the star is seen at a wavelength of $\lambda = 656.034 \text{ nm}$ in air. The radial velocity of the star is,

$$v_r = c \times \frac{\lambda - \lambda_0}{\lambda_0} = -1.13 \times 10^{-7} \text{ cm s}^{-1} = -113 \text{ km s}^{-1} \quad (19)$$

As per convention, a negative value of V_r implies that Barnard's star is approaching earth. Note that the velocity of earth in its orbit around the sun is 30 km s^{-1} , and cannot be neglected. This is why the velocities of stars are usually reported in the barycentric frame. The refractive index of air, $n = 1.000297$ only affects the speed of light by a slight value, and can be neglected in this case as all measurements are made in air.

b) Tangential velocity The parallax of Barnard's star is $p = 0.549''$ giving a distance $d = 1.82 \text{ pc}$. Then, we use the proper motion μ to calculate linear velocity:

$$v_t = d\mu = 1.82 \text{ pc} \times 10.4'' \text{ yr}^{-1} = 5.62 \times 10^{18} \text{ cm} \cdot 1.59 \times 10^{-12} \text{ rad yr}^{-1} = 89.4 \text{ km s}^{-1} \quad (20)$$

Alternately, we can use a conversion factor directly: a proper motion of $1'' \text{ yr}^{-1}$ corresponds to a linear velocity of 4.74 km s^{-1} at a distance of 1 pc . Thus, we get:

$$v_t = \frac{10.4''}{\text{yr}} \times \frac{4.74 \text{ km s}^{-1}}{1'' \text{ yr}^{-1} \text{ pc}} \times 1.82 \text{ pc} = 89.4 \text{ km s}^{-1} \quad (21)$$

c) Space velocity The total speed is simply obtained by vector adding the two perpendicular components of velocity. Thus we get $|v| = \sqrt{v_r^2 + v_t^2} = 144 \text{ km s}^{-1}$.

3 Sirius

a) Total mass of the system The parallax of the Sirius system is $0.38''$, giving a distance $d = 2.64 \text{ pc}$. Using the angular size $\theta = 7.61''$, we get that the semimajor axis of the system is $a = d \cdot \theta = 20 \text{ AU}$. We know the period is $P = 49.94 \text{ yr}$. Then we can use Equation 13 to calculate the total mass of the system:

$$M_A + M_B = \frac{a^3}{P^2} M_\odot = 3.24 M_\odot \quad (22)$$

The ratio of their masses is simply $\frac{M_A}{M_B} = \frac{a_B}{a_A}$. So we get the individual masses as $M_A = 2.2 M_\odot$, $M_B = 1.0 M_\odot$.

b) Luminosities For calculating the bolometric luminosities, we use the fact that the absolute bolometric magnitude of the sun is $m_\odot = 4.74$. Note that here we are representing absolute magnitude with m instead of M to avoid confusion with mass. We then use the relation,

$$L_* = L_\odot 10^{(m_* - m_\odot)/-2.5} \quad (23)$$

Using $m_A = 1.36$ we get $L_A = 22.5 L_\odot$. Similarly $m_B = 8.79$ gives $L_B = 0.024 L_\odot$.

c) Radius of Sirius B Sirius B has a very high temperature, $T_B = 24790 \text{ K}$. Using $L_\odot = 3.8 \times 10^{33} \text{ ergs cm}^{-2} \text{ s}^{-1}$ we get,

$$R_B = \sqrt{\frac{L_B}{4\pi\sigma T_B^4}} = 5.85 \times 10^8 \text{ cm} = 0.0084 R_\odot = 0.92 R_\oplus \quad (24)$$

Where as usual, R_\oplus is radius of the earth. The radius is so small as Sirius B is not a main sequence star, but a white dwarf.