I. NEUTRALINO DARK MATTER

It is possible that the lightest supersymmetric particle is a stable neutralino. It is natural then to consider the possibility that neutralinos are the dark matter.

A. Freeze out and WIMPs

Dark matter may be produced in a simple and predictive manner as a thermal relic of the Big Bang. The very early Universe is a very simple place – all particles are in thermal equilibrium. As the universe cools and expands, however, interaction rates become too slow to maintain this equilibrium, and so particles “freeze-out”. Unstable particles that freeze out disappear from the universe. However, the number of stable particles asymptotically approaches a non-vanishing constant, and this, their thermal relic density, survives to the present day.

This process is described quantitatively by the Boltzmann equation

$$\frac{dn}{dt} = -3Hn - \langle \sigma_A v \rangle (n^2 - n_{eq}^2),$$

where $n$ is the number density of the dark matter particles, $H$ is the Hubble constant, $\langle \sigma_A v \rangle$ is the thermally averaged annihilation cross-section, and $n_{eq}$ is the $\chi$ number density in thermal equilibrium. On the right-hand side of Eq. (1), the first term accounts for dilution from expansion. The $n^2$ term arises from processes $\chi\chi \rightarrow f\bar{f}$ that destroy $\chi$ particles, and the $n_{eq}^2$ term arises from the reverse process $f\bar{f} \rightarrow \chi\chi$, which creates $\chi$ particles.

It is convenient to change variables from time to temperature,

$$t \rightarrow x \equiv m/T,$$

where $m$ is the $\chi$ mass, and to replace the number density by the co-moving number density,

$$n \rightarrow Y \equiv n/s,$$

where $s$ is the entropy density. The expansion of the universe has no effect on $Y$, because $s$ scales inversely with the volume of the universe when entropy is conserved. In terms of these new variables, the Boltzmann equation is written as follows:

$$\frac{x}{Y_{eq}} \frac{dY}{dx} = -\frac{n_{eq} \langle \sigma_A v \rangle}{H} \left( \frac{Y^2}{Y_{eq}^2} - 1 \right).$$
In this form, it is clear that before freeze out, when the annihilation rate is large compared with the expansion rate, $Y$ tracks its equilibrium value $Y_{eq}$. After freeze-out, $Y$ approaches a constant. This constant is determined by the annihilation cross section $\langle \sigma_A v \rangle$. The larger this cross section, the longer $Y$ follows its exponentially decreasing equilibrium value, and the lower the thermal relic density.

Let us now consider WIMPs – weakly interacting massive particles with mass and annihilation cross section set by the weak scale: $m^2 \sim \langle \sigma_A v \rangle^{-1} \sim m_{weak}^2$. Freeze out takes place when

$$n_{eq} \langle \sigma_A v \rangle \sim H.$$  

Neglecting numerical factors, $n_{eq} \sim (mT)^{3/2}e^{-m/T}$ for a non-relativistic particle, and $H \sim T^2/M_*$. From these relations, we find that the WIMPs freeze out when

$$\frac{m}{T} \sim \ln \left( \frac{\langle \sigma_A v \rangle}{mM_*} \left( \frac{m}{T} \right)^{1/2} \right) \sim 30.$$  

Since $\frac{1}{2}mv^2 = \frac{3}{2}T$, WIMPs freeze out with velocity $v \sim 0.3$.

One might think that, since the number density of particles falls exponentially once the temperature drops below its mass, freeze out should occur at $T \sim m$. This is not the case. Because gravity is weak and $M_*$ is large, the expansion rate is extremely slow, and freeze out occurs much later than one might naively expect. For an $m \sim 300$ GeV particle, freeze out occurs not at $T \sim 300$ GeV and time $t \sim 10^{-12}$ s, but rather at temperature $T \sim 10$ GeV and time $t \sim 10^{-8}$ s.

With a little more work, one can find not just the freeze out time, but also the freeze out density. A rough and intuitive derivation goes as follows. The freeze out occurs when the rate of the annihilation cross section is similar to the expansion rate of the universe,

$$\Gamma \sim n\langle \sigma_A v \rangle \lesssim H \sim T^2/M_*.$$  

Taking into account that $T \propto m$ [see Eq. (6)], we have

$$\frac{n_{DM}}{n_\gamma} \propto \frac{m^2/M_* \langle \sigma_A v \rangle}{m^3} \sim \frac{1}{M_* \langle \sigma_A v \rangle m},$$  

so that

$$\frac{\rho_{DM}(T)}{\rho_\gamma(T)} \sim \frac{m n_{DM}}{T n_\gamma} \propto \frac{1}{M_* \langle \sigma_A v \rangle T}.$$  


Inserting the value of $\rho_{\text{DM}}/\rho_c$ at $T_0$ one obtains $\langle \sigma A v \rangle \sim T_0 M_* \sim (\text{TeV})^2$ or, more accurately,

$$\Omega_{\chi} = m_{\chi} Y(x = \infty) \sim \frac{10^{-10} \text{ GeV}^{-2}}{\langle \sigma A v \rangle}. \quad (10)$$

It is an intriguing fact that a typical weak cross-section,

$$\langle \sigma A v \rangle \sim \frac{\alpha^2}{m_{\text{weak}}} \sim 10^{-9} \text{ GeV}^{-2}, \quad (11)$$

leads to a thermal relic density of $\Omega h^2 \sim 0.1$. WIMPs therefore naturally have thermal relic densities of the observed magnitude. The above analysis has ignored many numerical factors, and the thermal relic density may vary by as much as a few orders of magnitude. Nevertheless, in conjunction with the other strong motivations for new physics at the weak scale, this coincidence is an important hint that the problems of electroweak symmetry breaking and dark matter may be intimately related.

We now want to apply the general formalism above to the specific case of neutralinos. This is complicated by the fact that neutralinos may annihilate to many final states: $f \bar{f}, W^+ W^-, ZZ, Zh, hh$, and states including the heavy Higgs bosons $H, A$ and $H^\pm$. Many processes contribute to each of these final states, and nearly every supersymmetric parameter makes an appearance in at least one process. Given this complicated picture, it is not surprising that there are a variety of ways to achieve the desired relic density for neutralino dark matter. What is surprising, however, is that many of these different ways may be found in minimal supergravity, provided one looks hard enough. We therefore consider various regions of minimal supergravity parameter space where qualitatively distinct mechanisms lead to neutralino dark matter with the desired thermal relic density.

**B. Bulk Region**

The LSP is a Bino-like neutralino in much of mSUGRA parameter space. It is useful, therefore, to begin by considering the pure Bino limit. In this case, all processes with final state gauge bosons vanish. This follows from supersymmetry and the absence of 3-gauge boson vertices involving the hypercharge gauge boson.

The process $\chi \chi \rightarrow f \bar{f}$ through a $t$-channel sfermion does not vanish in the Bino limit. This reaction has an interesting structure. Recall that neutralinos are Majorana fermions. If the initial state neutralinos are in an $S$-wave state, the Pauli exclusion principle implies
that the initial state is CP-odd with total spin $S = 0$ and total angular momentum $J = 0$. If the neutralinos are gauginos, the vertices preserve chirality, and so the final state $f \bar{f}$ has spin $S = 1$. This is compatible with $J = 0$ only with a mass insertion on the fermion line. This process is therefore either $P$-wave suppressed (by a factor $v^2 \sim 0.1$) or chirality suppressed (by a factor $m_f/m_W$). In fact, this conclusion holds also for mixed gaugino-higgsino neutralinos and for all other processes contributing to the $f \bar{f}$ final state.

The region of mSUGRA parameter space with a bino-like neutralino where $\chi \chi \rightarrow f \bar{f}$ yields the right relic density is the $(m_0, m_{1/2}) \sim (100 \text{ GeV}, 200 \text{ GeV})$ region. It is called the “bulk region” as, in the past, there was a wide range of parameters with $m_0, m_{1/2} \lesssim 300 \text{ GeV}$ that predicted dark matter within the observed range. The dark matter energy density has by now become so tightly constrained, however, that the “bulk region” has now been reduced to a thin ribbon of acceptable parameter space.

Moving from the bulk region by increasing $m_0$ and keeping all other parameters fixed, one finds too much dark matter. This behavior is not difficult to understand: in the bulk region, a large sfermion mass suppresses $\langle \sigma_A v \rangle$ which implies a large $\Omega_{\text{DM}}$. In fact, sfermion masses not far above current bounds are required to offset the $P$-wave suppression of the annihilation cross section. This is an interesting fact – cosmology seemingly provides an upper bound on superpartners masses! If this were true, one could replace subjective naturalness arguments by the fact that the universe cannot be overclosed to provide upper bounds on sparticle masses.

Unfortunately, this line of reasoning is not airtight even in the CMSSM. The discussion above assumes that $\chi \chi \rightarrow f \bar{f}$ is the only annihilation channel. In fact, however, for non-bino-like neutralinos, there are many other contributions. Exactly this possibility is realized in the focus point region, which we describe next.

In passing, it is important to note that the bulk region, although the most straightforward and natural in many respects, is also severely constrained by other data. The existence of light superpartner spectrum in the bulk region implies a light Higgs boson mass, and typically significant deviations in low energy observables such as $b \rightarrow s\gamma$ and $(g-2)_\mu$. Current bounds on the Higgs boson mass and standard model predictions for $b \rightarrow s\gamma$ and (possibly) $(g-2)_\mu$ disfavor this region. For this reason, it is well worth considering other possibilities, including the three we now describe.
C. Focus point region

A bino-like LSP is not a definite prediction of the CMSSM. For large scalar mass parameter \( m_0 \), the higgsino mass parameter \(|\mu|\) drops to accommodate electroweak symmetry breaking. The LSP then becomes a gaugino-higgsino mixture. The region where this happens is called the focus point region, a name derived from peculiar properties of the renormalization group equations which suggest that large scalar masses do not necessarily imply fine-tuning.

In the focus point region, the \( t \)-channel \( \tilde{f} \)-mediated \( \chi \chi \rightarrow f \bar{f} \) contribution is suppressed by very heavy sfermions. However, the existence of higgsino components in the LSP implies that \( t \)-channel chargino-mediated \( \chi \chi \rightarrow W^+W^- \) diagrams are no longer suppressed. This provides a second method by which neutralinos may annihilate efficiently enough to produce the desired thermal relic density. The right amount of dark matter can be achieved with arbitrarily heavy sfermions, and so there is no useful cosmological upper bound on sparticle masses, even in the framework of mSUGRA.

D. A-funnel region

A third possibility realized in mSUGRA is that the dark matter annihilates to fermion pairs through an \( s \)-channel pole. The potentially dominant process is through the \( A \) Higgs boson. The \( A \) is CP-odd, and so may couple to an initial \( S \)-wave. This process is efficient when \( m_A \approx 2m_\chi \). In fact, the \( A \) resonance may be broad, extending the region of parameter space over which this process is important.

The \( A \) resonance region occurs in the CMSSM for \( \tan \beta \gtrsim 40 \). The resonance is so efficient that the relic density may be reduced too much. The desired relic density is therefore obtained when the process is near resonance, but not exactly on it.

E. Co-annihilation region

The desired neutralino relic density may be obtained even if \( \chi \chi \) annihilation is inefficient if there are other particles present in significant numbers when the LSP freezes out. The neutralino density may then be brought down through co-annihilation with the other species. Naively, the presence of other particles requires that they be mass degenerate with the
neutralino to within the temperature at freeze out, $T \approx m_\chi/30$. In fact, co-annihilation may be so enhanced relative to the $P$-wave-suppressed $\chi\chi$ annihilation cross section that co-annihilation may be important even with mass splittings much larger than $T$.

The co-annihilation possibility is realized in mSUGRA along the $\tilde{\tau}$-LSP – $\chi$-LSP border. Processes such as $\chi\tilde{\tau} \rightarrow \tau^* \rightarrow \tau\gamma$ are not $P$-wave suppressed, and so enhance the $\chi\chi$ annihilation substantially. There is therefore a narrow finger extending up to masses $m_\chi \sim 600 \text{ GeV}$ with acceptable neutralino thermal relic density.

II. GRAVITINO COSMOLOGY

In the previous section, we ignored the gravitino. In this section, we rectify this omission. Although gravitino interactions are highly suppressed, gravitinos may have implications for many aspects of cosmology, including big bang nucleosynthesis (BBN), the cosmic microwave background (CMB), inflation, and reheating. Gravitino cosmology is in many ways complementary to neutralino cosmology, providing another rich arena for connections between microscopic physics and cosmology.

A. Gravitino properties

The properties of gravitinos may be systematically derived by supersymmetrizing the standard model coupled to gravity. Here we remind the reader of the highlights of this analysis.

In an exactly supersymmetric theory, the gravitino is a massless spin-3/2 particle with two degrees of freedom. Once supersymmetry is broken, the gravitino eats a spin-1/2 fermion, the Goldstino, and becomes a massive spin-3/2 particle with four degrees of freedom. The resulting gravitino mass is $m_\tilde{G} = F/\sqrt{3}M_*$, where $M_* = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck mass. Gravitinos couple standard model particles to their superpartners through gravitino-gaugino-gauge boson interaction,

$$\mathcal{L} = -\frac{i}{8M_*} \tilde{G}_\mu [\gamma^\nu, \gamma^\rho] \gamma^\mu \lambda F_{\nu\rho},$$

and gravitino-fermion-sfermion interactions,

$$\mathcal{L} = -\frac{1}{\sqrt{2}M_*} \partial_\nu \tilde{f} \tilde{f} \gamma^\nu \tilde{G}_\mu.$$

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In models with high scale supersymmetry breaking, such as conventional supergravity theories, \( F \sim m_{\text{weak}} M_\ast \). The gravitino mass is therefore of the order of the other sparticle masses, and we expect them all to be in the range \( \sim 100 \text{ GeV} - 1 \text{ TeV} \). The gravitino effective couplings are \( \sim E/M_\ast \), where \( E \) is the energy of the process. The gravitino interactions are therefore typically extremely weak, as they are suppressed by the Planck scale.

We focus here on theories with high-scale supersymmetry breaking. Note, however, that in theories with low-scale supersymmetry breaking, the gravitino may be much lighter, for example, as light as \( \sim \text{eV} \) in some simple GMSB models. Its interactions through its Goldstino components may also be much stronger, suppressed by \( F/m_{\text{weak}} \) rather than \( M_\ast \).

**B. Thermal relic density**

If gravitinos are to play a cosmological role, we must first identify their production mechanism. There are a number of possibilities. A natural starting point is to consider gravitino production as a result of freeze out from thermal equilibrium. At present, the gravitino coupling \( E/M_\ast \) is a huge suppression. However, if we extrapolate back to very early times with temperature \( T \sim M_\ast \), even gravitational couplings were strong, and gravitinos were in thermal equilibrium, \( n_{\tilde{G}} = n_{\text{eq}} \). Once the temperature drops below the Planck mass, however, gravitinos quickly decouple with the number density appropriate for relativistic particles. Following decoupling, their number density then satisfies \( n_{\tilde{G}} \propto R^{-3} \propto T^3 \). This has the same scaling behavior as the background photon number density, so we expect roughly similar number densities at present.

If such gravitinos are stable, they could be the dark matter. However, the overclosure bound implies

\[
\Omega_{\tilde{G}} \lesssim 1 \quad \Rightarrow \quad m_{\tilde{G}} \lesssim 1 \text{ keV}. \tag{14}
\]

This is not surprising – relic neutrinos have a similar density, and the overclosure bound on their mass is similar.

On the other hand, gravitinos may be unstable. This may be because \( R \)-parity is violated, or because the gravitino is not the LSP. In this case, there is no bound from overclosure, but there are still constraints. In particular, the decay products of the gravitino can destroy the successful predictions of BBN for light element abundances if the decay takes place after BBN. In the case where the decay to lighter supersymmetric particle is possible, we can
estimate the gravitino lifetime to be
\[ \tau_{\tilde{G}} \sim \frac{M^2}{m^2_{\tilde{G}}} \sim 0.1 \text{ yr} \left( \frac{100 \text{ GeV}}{m_{\tilde{G}}} \right)^3. \] (15)

Requiring gravitino decays to be completed before BBN at \( t \sim 1 \text{ s} \) implies
\[ m_{\tilde{G}} \gtrsim 10 \text{ TeV}. \] (16)

In both cases, the required masses are incompatible with the most natural expectations of conventional supergravity theories. Gravitinos may, however, be a significant component of dark matter if they are stable with mass \( \sim \text{ keV} \). Such masses are possible in low-scale supersymmetry breaking scenarios (with \( F \sim \text{ keV} \times M_* \sim (10^6 \text{ GeV})^2 \)).

C. Production during reheating

In the context of inflation, the gravitino production mechanism of the previous subsection is rather unnatural. Between the time that \( T \sim M_* \) and now, we expect the universe to inflate, which would dilute any gravitino relic thermal density. Inflation does provide another source for gravitinos, however. Specifically, following inflation, we expect an era of reheating, during which the energy of the inflaton potential is transferred to standard model particles and superpartners, creating a hot thermal bath in which gravitinos may be produced.

After reheating, the universe is characterized by three hierarchically separated rates: the interaction rate of the MSSM particles with each other \( \sigma_{\text{SM}n} \); the expansion rate \( H \); and the rate of interactions involving the gravitino \( \sigma_{\tilde{G}n} \). Here \( n \) is the number density of standard model particles. After reheating, the universe is expected to have a temperature well below the Planck scale, but well above the weak scale. These rates may then be estimated by dimensional analysis, and we find
\[ \sigma_{\text{SM}n} \sim T \gg H \sim \frac{T^2}{M_*} \gg \sigma_{\tilde{G}n} \sim \frac{T^3}{M_*^2}. \] (17)

The picture that emerges then is that after reheating, there is a thermal bath of MSSM particles. Occasionally, these interact to produce a gravitino through interactions like \( gg \rightarrow g\tilde{G} \). The produced gravitinos then propagate through the universe essentially without interacting. If they are stable, as we assume in this section, they contribute to the present dark matter density.
To determine the gravitino abundance, we turn once again to the Boltzmann equation:

\[ \frac{dn}{dt} = -3Hn - \langle \sigma_A v \rangle (n^2 - n_{\text{eq}}^2). \]  

(18)

In this case, the source term \( n_{\text{eq}}^2 \) arises from interactions such as \( gg \rightarrow \tilde{g}\tilde{G} \). In contrast to our previous application of the Boltzmann equation, however, here the \( n^2 \) sink term, originating from interactions such as \( \tilde{g}\tilde{G} \rightarrow gg \), is negligible. Changing variables as before, \( t \rightarrow T \) and \( n \rightarrow Y \equiv n/s \), we obtain

\[ \frac{dY}{dT} = -\frac{\langle \sigma_{\tilde{G}v} \rangle}{HTs} n^2. \]  

(19)

The right hand side is independent of \( T \), since \( n \propto n^3 \), \( H \propto T^2 \) and \( s \propto T^3 \). We thus find an extremely simple relation – the gravitino relic number density is linearly proportional to the reheat temperature \( T_R \).

The constant of proportionality is the gravitino production cross section. The leading \( 2 \rightarrow 2 \) QCD interactions have been calculated, so that the gravitino relic density can be determined as a function of \( T_R \) and \( m_{\tilde{G}} \). For \( m_{\tilde{G}} \sim 100 \text{ GeV} \), the constraint on \( \Omega_{\text{DM}} \) requires

\[ T_R \lesssim 10^{10} \text{ GeV}, \]  

(20)

providing a bound on the inflaton potential. Of course, if this bound is nearly saturated, gravitinos produced after reheating may be a significant component of dark matter.

D. Production from late decays

A third mechanism for gravitino production is through the cascade decays of other sparticles. If the gravitino is not the LSP, cascade decays will bypass the gravitino, given its highly suppressed coupling. However, the gravitino might be the LSP, even in high-scale supersymmetry breaking models. If the gravitino is the LSP, all cascades will ultimately end in a gravitino.

An alternative gravitino dark matter scenario is therefore the following. Assume that the gravitino is the LSP and stable. To separate this scenario from the previous two, assume that inflation dilutes the primordial gravitino density and the universe reheats to a temperature low enough that gravitino production is negligible. Because the gravitino couples only gravitationally with all interactions suppressed by Planck scale, it plays no role in the thermodynamics of the early universe. The next-to-lightest supersymmetric particle
(NLSP) therefore freezes out as usual. If it is weakly interacting, its relic density will be near the desired value. However, much later, after
\[ \tau \sim \frac{M_*^2}{m_{\text{weak}}^3} \sim 10^5 \text{s} - 10^8 \text{s}, \]
the NLSP decays to the gravitino LSP. The gravitino therefore becomes dark matter with relic density
\[ \Omega_{\tilde{G}} = \frac{m_{\tilde{G}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}}. \]
The gravitino and NLSP masses are naturally of the same order in theories with high scale supersymmetry breaking. Gravitino LSPs may therefore form a significant relic component of our universe, inheriting the desired relic density from WIMP decay. In contrast to the previous two production mechanisms, the desired relic density is achieved naturally without the introduction of new energy scales.

The decay time of Eq. (21), well after BBN, should be of concern. The decaying particle is a WIMP, and so has a density far below that of a relativistic particle. However, one must check to see if the light element abundances are greatly perturbed. In fact, for some weak scale NLSP and gravitino masses they are, and for some they are not.

Models with weak scale extra dimensions also provide a similar dark matter particle in the form of Kaluza-Klein gravitons, with Kaluza-Klein gauge bosons or leptons playing the role of the decaying WIMP. Because such dark matter candidates naturally preserve the WIMP relic abundance, but have interactions that are weaker than weak, they have been named superweakly interacting massive particles, or “super-WIMPs”.

Our discussion of WIMP (neutralino) dark matter was only valid for the “half” of the parameter space where \( m_{3/2} > m_{\text{LSP}} \). When the gravitino is the LSP, there are number of new implications of supersymmetry for cosmology. For example, the “\( \tilde{\tau} \) LSP” region is no longer excluded by searches for charged dark matter, as the \( \tilde{\tau} \) is no longer stable, but only metastable. There is therefore the possibility of stable heavy charged particles appearing in collider detectors. Further, regions with too much dark matter are no longer excluded, because the gravitino dark matter density is reduced by \( m_{3/2}/m_{\text{NLSP}} \) relative to the NLSP density. The late decays producing gravitinos may have detectable consequences for BBN and CMB. Astrophysical signatures in the diffuse photon spectrum, the ionization of the
universe, and the suppression of small scale structure are also interesting possibilities.