Physics 106b – Problem Set 11 – Due Mar 2, 2005

Version 1

February 6, 2005

These problems cover the material on special relativity in Hand and Finch Chapter 12 and Section 6.1 of the lecture notes.

This problem set is due on Wed Mar 2 at 5 pm (1 week before the last day of classes) at 311 Downs. It will be worth 50% of a normal problem set. Late problem sets may be turned in up to 1 week late for 50% credit as usual. You have **plenty** of lead time on this problem set, the fact that it is due one day after the Ph125 set will not be accepted as a mitigating factor.

The original intention was to include these on the first E & M problem set, but that did not happen. You no doubt will be unhappy about this extra work, but it would be negligent to have assigned no problems on special relativity – as we always say, the only way to understand the material is to do problems. You'll be thankful if you take the physics GRE[1] or any kind of serious astrophysics or particle/nuclear physics classes (and certainly if you take relativistic QM).

I will not be holding regular office hours, but feel free to arrange a special appointment with me if you need help with the material.

1. Hand and Finch 12.6 (relativistic acceleration). This problem is very useful for calculating the motion of a charged particle in various accelerators (cyclotron, synchrotron, linear accelerator). In addition to what is asked, do the following:

   (a) Show that

   $$|a^\mu|^2 = \frac{\gamma^6}{\gamma_\perp^2}\left|\frac{d\vec{\beta}}{dt}\right|^2$$

   where

   $$\gamma_\perp^2 = \left(1-\beta_\perp^2\right)^{-1} = \left(1-\left[\beta^2-\left(\vec{\beta}\cdot\frac{\frac{d\vec{\beta}}{dt}}{\left|\frac{d\vec{\beta}}{dt}\right|}\right)^2\right]\right)^{-1}$$

   is the γ factor due to the lab-frame velocity perpendicular to the lab-frame acceleration. (It's less complicated than it looks.)

   (b) Show that the above formula reduces to the two that you found for circular motion and parallel acceleration.

[1] see, for example, **http://128.148.60.98/physics/userpages/students/Artur_Adib/gre.html** or just google "physics GRE relativity")

Notes:

- As in the definition of four-velocity, you will find it necessary to convert $\frac{d}{d\tau}$ to $\frac{d}{dt}$ to obtain the desired explicit form. In our demonstration that $u^\mu = \gamma\left(1, \vec{\beta}\right)$, we used $\tau = \sqrt{|x^\mu|^2}$ and took the derivative with respect to t. We implicitly assumed there that $\vec{\beta}$ was constant. Since we are no longer making that assumption, the original relation for $\tau(t)$ may no longer hold. What we can be sure of, though, is that the infinitesimal version of the relation holds:

$$\begin{aligned}(d\tau)^2 &= (dt)^2 - (d\vec{x} \cdot d\vec{x})^2 \\ d\tau &= dt\sqrt{1-\beta^2} = \frac{dt}{\gamma}\end{aligned}$$

  where $\vec{\beta}$ may now be a function of time. You will want to use the differential relation.[2]

- There is a minor error in the problem: it should ask you to demonstrate that "the relation between the laboratory acceleration of a particle undergoing circular motion at constant speed and the acceleration in the instantaneous rest frame of the particle is $a_{rest} = \gamma^2 \, a_{lab}$." That this is a typo is confirmed by Goldstein derivation 7.7 and the generic formula for $|a^\mu|^2$ given above.

2. Hand and Finch 12.17 (Compton scattering). Notes:

   (a) In class we have not used the notation $k_\mu \, k^\mu$ or $p_\mu \, p^\mu$. In general, for any four-vector, the expression $a_\mu \, a^\mu$ is the invariant norm $|a^\mu|^2 = (a^0)^2 - (a^1)^2 - (a^2)^2 - (a^3)^2$. The formal meaning of lowered indices is discussed in the lecture notes, but you don't need to know that for this problem.

   (b) The k^μ defined in this problem is different from the k^μ defined in class by a factor $\hbar$. You can basically forget about the k^μ defined in class when doing this problem – just take it as given that k^μ is the four-momentum of the photon, with the time component being the energy and the space components being the spatial momentum, and that $|k^\mu|^2 = 0$ because the photon rest mass vanishes.

   (c) A "backscattered" photon is one with outgoing angle $\theta' = \pi$.

3. You should look at and understand conceptually how to do Hand and Finch 12.4 (the pole-in-barn problem), but **it is not to be turned in**. This is a classic "relativity of simultaneity" problem that everyone ought to know how to do.

[2] Two finer points: 1) with the differential relation, we could of course have derived the four-velocity expression without the assumption of constant velocity; hence, it holds even when the velocity is not constant. 2) If we integrate the differential relation, we find

$$\tau(t) = \int_0^t d\tilde{t}\sqrt{1 - \left[\beta(\tilde{t})\right]^2}$$

which may in general be different from $\sqrt{|x^\mu|^2} = \sqrt{t^2 - \vec{x}\cdot\vec{x}}$, hence the distinction between using the differential and integral relations.