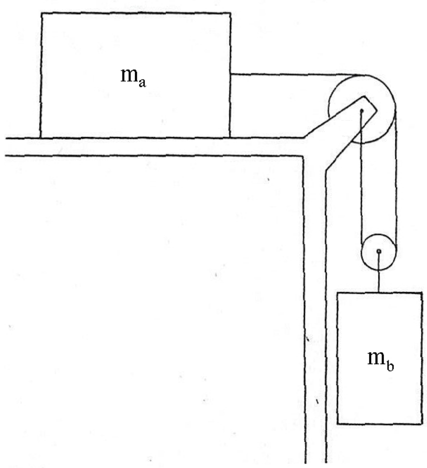
Physics 106a/196a – Problem Set 1 – Due Oct 7, 2005

These problems cover the material on Newtonian mechanics and gravitation in Sections 1.1 and 1.2 of the lecture notes. Problems 1 and 2 are for 106a students only, 3, 4, and 5 for 106a and 196a students, and 6 and 7 for 196a students only.

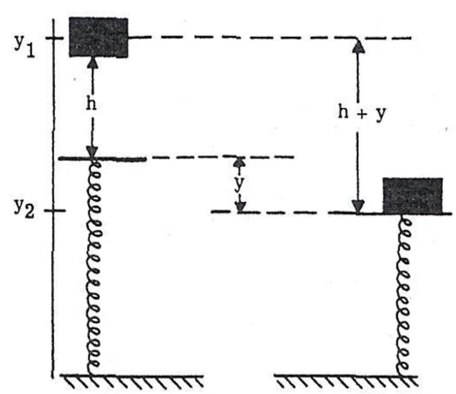
You are reminded that:

- The force exerted by a spring is $F = -k\,(x - x_0)$ where x is the length of the spring and x_0 is the rest length of the spring; the force is opposite the displacement from the rest length. In vector notation, $\vec{F} = -k\,(\vec{r} - \vec{r}_0)$.
- The potential energy function of a spring is $U(x) = \frac{1}{2}\,k\,(x - x_0)^2$, or $U(\vec{r}) = \frac{1}{2}\,k\,|\vec{r} - \vec{r}_0|^2$.

1. (106) Consider the following system of two masses m_a and m_b and pulleys. m_a slides frictionlessly on the table. m_b is suspended from a massless pulley. Both pulleys are frictionless, and the two ropes are massless and inextensible. Find the accelerations of the two masses and the tensions in the two ropes.



Problem 1

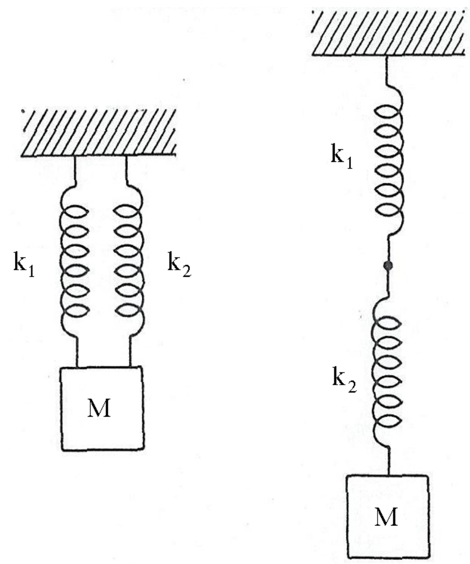


Problem 2

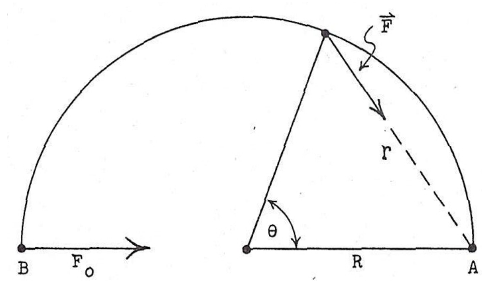
2. (106) A block of mass m subject to gravity is dropped from a height h onto a massless spring with spring constant k. Find the maximum distance that the spring will be compressed using energy methods. (See diagram for definition of h.)

3. (106/196) Two springs of different spring constants k_1 and k_2 are connected either in parallel or series to a mass M. M is also subject to gravity. Both systems are in equilibrium – there is no net acceleration of M. For each case, find the extensions of the two springs and the effective spring constant (the spring constant of a single spring that would result in the same

behavior). Denote the rest lengths of the springs by z_{10} and z_{20}; the rest lengths may be different in the parallel and series cases. Hint: What is the same for the two springs in the parallel case? In the series case?



Problem 3



Problem 4

4. (106/196) A particle moves 180° around a semicircular path of radius R. It is attracted toward its starting point A by a force proportional to its distance from A. At the final point, B, this force towards A is F_0. Calculate the work done against this force when the particle moves around the semicircle from A to B. Note that the force is that of a simple spring of rest length 0. You may use the spring potential energy function to check your result, but you *must* do the problem by calculating the work integral explicitly; the problem is intended to give you practice in this.

5. (106/196) This problem helps explain the mass distribution in globular clusters, which are nearly spherically symmetric clusters of stars. Suppose a very large number N of point objects are all moving under their mutual gravitational attraction. All objects have equal masses m and equal kinetic energies E and therefore equal speeds v. Each moves in a circular orbit around the common center-of-mass of the system. N is large enough so that the mass density $\rho(r)$ can be considered continuous. Find $\rho(r)$. (Hint: Start by finding the force acting on a star at radius r to keep it moving in its circular orbit. How is this force related to the mass density?).

6. (196) A car is driving through a banked curve. The bank angle is θ and the curve radius is R. For what speed is the curve ideally banked?

7. (196) The earth has approximately the shape of an oblate ellipsoid of revolution whose polar diameter $2\,a\,(1-\eta)$ is slightly shorter than its equatorial diameter $2\,a$. ($\eta = 0.0034$) To determine to first order in η the effect of the earth's oblateness on its gravitational field, we may replace the ellipsoidal earth by a sphere of radius R so chosen as to have the same volume. The gravitational field of the earth is then the field of a uniform sphere of radius R with the mass of the earth, plus the field of a surface distribution of mass (positive or negative), representing the mass per unit area which would be added or subtracted to form the actual ellipsoid.

(a) Show that the required surface density is, to first order in η,

$$\sigma(\theta, \phi) = \frac{1}{3}\,\eta\, a\, \rho\, \left(1 - 3\,\cos^2\theta\right)$$

where θ is the colatitude (polar angle) and ρ is the volume density of the earth (assumed uniform). Since the total mass thus added to the surface is zero, its gravitational field will represent the effect of the oblate shape of the earth.

(b) Show that the resulting correction to the gravitational potential at a very great distance $r \gg a$ is, to order a^3/r^3,

$$\delta\Psi(r, \theta, \phi) = \frac{1}{5}\,\eta\,\frac{M\,G\,a^2}{r^3}\,\left(1 - 3\,\cos^2\theta\right)$$

Hint: You will of course have to Taylor expand the relative distance $|\vec{r}_2 - \vec{r}_1|$ in the denominator of the usual expression for the potential. You will need to go beyond a first-order expansion (as is evidenced by the presence of r^3). You will not be penalized if you are unable to get the numerical prefactors correct in (b).