Physics 106b – Problem Set 10 – Due Jan 27, 2006

Version 2 – Jan 23, 2006

This set covers dynamics in rotating systems and some of the early material on rigid body motion, Sections 5.1, 5.2, and 5.3.1 of the lecture notes and Chapters 7 and 8 of Hand and Finch. Be sure to have checked the lecture notes errata! Problems 1 through 4 are required, problem 5 is extra credit and equal in weight to the first four problems.

Changes since v.1: Clarifications on problems 3 and 4b.

1. A particle is thrown vertically upward with initial speed v_0, reaches a maximum height, and falls back to the ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four time greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.

2. Prove Larmor's Theorem: Consider a system of charged particles, all having the same ratio q/m of charge to mass, that experience mutual central forces $\vec{F}_{ab}(\vec{r}_{ab})$ and are also subject to a conservative external force $\vec{F}_e(\vec{r}_a)$. Show that, if the system is subjected to a *weak*, uniform, constant magnetic field $\vec{B}$, then one can eliminate the effect of the field by observing the motion of the system in a coordinate system that is rotating relative to the initial inertial frame with angular velocity

$$\vec{\omega} = -\frac{q}{2\,m\,c}\,\vec{B}$$

(in Gaussian units). Be sure to specify what is meant by "weak". You may *not* look at Goldstein or Symon!

3. In Section 5.2.3 of the lecture notes, we discuss the Lagrangian and Hamiltonian approaches to the Foucault pendulum problem. We neglected the ω^2 term in the kinetic energy. Repeat the calculation of the Lagrangian and Hamiltonian with the ω^2 term, but setting $\lambda = \frac{\pi}{2}$ at the beginning of the calculation. You should start with the equation from the notes

$$T = \frac{1}{2}\,m\left(\left(\dot{x}^2 + \dot{y}^2\right) + 2\,\omega\,\sin\lambda\,\left(-y\,\dot{x} + x\,\dot{y}\right) + \omega^2\,\sin^2\lambda\,\left(x^2 + y^2\right) + \omega^2\,\cos^2\lambda\,x^2\right)$$

Show that one obtains an equation of motion that can be derived from an effective one-dimensional potential that is the same as we would have calculated based on our study of central forces. You will of course want to convert to cylindrical coordinates. Don't forget the subtleties about going from the 3-dimensional to the 1-dimensional Lagrangian. The canonical momentum in ϕ picks up an extra term here, what is the significance of that term?

4. Consider a uniform right circular cone of height h, half-angle α, mass M, and constant density.

   (a) Show that the inertia tensor of the cone in a coordinate system with the cone's axis along the z axis and the apex of the cone at the origin is

$$\underline{\mathcal{I}} = M\,h^2 \begin{pmatrix} \frac{3}{20}\left(4 + \tan^2\alpha\right) & 0 & 0 \\ 0 & \frac{3}{20}\left(4 + \tan^2\alpha\right) & 0 \\ 0 & 0 & \frac{3}{10}\,\tan^2\alpha \end{pmatrix}$$

(b) The cone rolls on its side without slipping on a uniform horizontal plane in such a manner that it returns to its original position in a time τ. Find expressions for the components of the angular velocity and angular momentum of the cone in the body and space frames and for the kinetic energy *in terms of the given parameters.* You may leave any inertia tensor factors in terms of the principal moments I_1 and I_3 rather than plugging in the above messy expressions for them. Note: this problem is intended to make sure you understand angular velocity and can calculate angular momentum and kinetic energy of a body undergoing rigid body motion. You do *not* need to use Euler's equations or calculate the torques in the problem (though, of course you may if you would enjoy doing so.)

5. In New Orleans (30° N latitude), there was a hockey arena with frictionless ice. The ice was formed by flooding a rink with water and allowing it to freeze slowly. This implies that a plumb bob would always hang in a direction perpendicular to the small patch of ice directly beneath it. Show that a hockey puck (shot slowly enough that it stays in the rink!) will travel in a *circle*, making one revolution every day. (Yes, this problem is somewhat trickier than it may appear at first glance . . .)