# Physics 106b – Problem Set 11 – Due Feb 3, 2006

Version 3 – Jan 31, 2006

This set covers rigid body motion, Section 5.3 of the lecture notes and Chapter 8 of Hand and Finch. Be sure to have checked the lecture notes errata! Problems 1 through 4 are required, problem 5 is extra credit and equal in weight to the first four problems.

Changes since v. 1: An independent source has confirmed that Hand and Finch have the incorrect sign for Problem 1. Clarifications in problem 3, numerical value for $\theta$ provided.

Changes since v. 2: Further clarifications on Problem 3.

1. Hand and Finch 8-15. The sign given for the result of part (c) appears to be incorrect; the result should be

$$\vec{\tau} = \frac{M\,a\,b\,\omega^2\left(b^2 - a^2\right)}{12\left(a^2 + b^2\right)}\,\hat{k}$$

2. Hand and Finch 8-21. When Hand and Finch say "Setting a certain determinant equal to zero gives you $\alpha$", realize that all they are asking you to do is to solve the set of three coupled linear equations. Be sure to try verifying the result experimentally! No looking at Thornton Section 11.12.

3. A proposed balloon payload to observe the polarization anisotropy of the cosmic microwave background will consist of a rotating gondola on which resides six refracting telescopes with rotating half-wave plates (HWP).[1] See the figure below. Each HWP is a disk-shaped object with its azimuthal symmetry axis aligned with the optical axis of its corresponding telescope, and hence it will be tilted relative to the vertical. It will will be suspended by a frictionless magnetic levitation bearing that will allow it to rotate about its symmetry axis. To prevent spurious signals, however, the bearing must precisely maintain the HWP's orientation as the gondola rotates and the HWP spins.

   We will consider the motion of and forces and torques acting on a single HWP. Because the HWP is sitting in a rotating gondola, it suffers from two kinds of disturbances. First, treated as a point particle, it suffers from the fictitious forces associated with being in the rotating gondola frame. Second, treated as a rigid body rotating about its symmetry axis (on which its center-of-mass sits), it also suffers from fictitious torques because it is being made to spin in the rotating gondola frame. Equivalently, in an inertial frame, it has both a spin angular velocity (relative to the gondola) and a precession angular velocity (due to the gondola's rotation). Determine how much force and torque must be exerted by the magnetic

[1] A half-wave plate is made of a birefringent material – one that has different indices of refraction for light polarized parallel to two different crystal axes. If one makes such a plate the correct thickness, light of one polarization travels half a wavelength more than light of the other polarization, resulting in a relative phase shift of $\pi$. This causes the polarization of the incoming light to rotate by an amount that depends on the angle between the incoming polarization angle and the crystal axes. In our case, sapphire will be used as the material. See the nice discussion at http://en.wikipedia.org/wiki/Wave_plate.

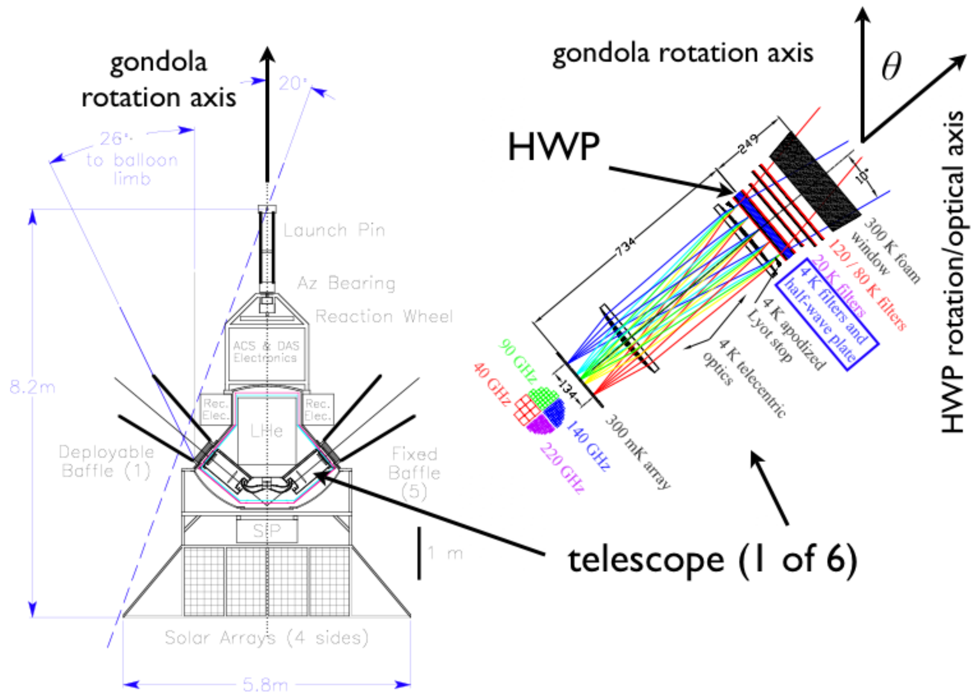


Figure 1: Problem 3. Left: gondola with telescope(s) sitting in it. Right: closeup on telescope showing half-wave plate.

levitation bearing to cancel these fictitious forces and torques. *Be sure to calculate the force and torque needed in the frame rotating with the gondola, not the frame spinning with the HWP. Experimentally, we apply the forces and torques in the gondola frame.*

You should assume the HWP has mass $M$, radius $R$, and thickness $t$ (it is just a disk of sapphire). The HWP has spin angular velocity $\Omega$ along its 3-axis. The telescope optical axis (and hence the HWP axis) makes an angle $\theta$ with the vertical. The gondola rotates about the vertical with angular speed $\omega_P$. Calculate the necessary forces for both steady motion and during the time $T_{accel}$ over which the gondola accelerates from rest to the full rotation rate, assuming uniform acceleration. Calculate the torque only for steady motion at the final rotation rate $\omega_P$. (If you are worried about it, you may assume the HWPs have been spun up to their spin speed $\Omega$ before any gondola rotation occurs.) Convert your calculated torque to a force that must be applied to the HWP, assuming the force is exerted at the HWP's outer edge and perpendicular to the plane of the HWP. Obtain numerical results using the following numbers:

| | |
|---|---|
| radius, $R$ | 15 cm |
| thickness, $t$ | 1 cm |
| density, $\rho$ | 4 g/cm$^3$ |
| distance from gondola axis to HWP center-of-mass, $r$ | 1.5 m |
| $\theta$ | 45 degrees |
| HWP spin rate, $\Omega$ | 10 revolutions/sec |
| gondola rotation rate, $\omega_P$ | 1 revolution/minute |
| time to accelerate gondola from rest to full rotation rate, $T_{accel}$ | 10 seconds |

4. Hand and Finch 8-24.

5. (Extra credit) Hand and Finch 8.18, but ignore the last two sentences:

> Is the gyroscope stable with respect to its original motion? The answer explains why gyroscopes are used to stabilize ships or video cameras, for example.

You will show in fact that the gyroscope is not stable with respect to its original polar angle $\theta_0$, but that it is stable about a new equilibrium polar angle $\theta_*$. The answer has nothing to do with how gyroscopic stabilization works.[2]

Do not obtain full solutions for the time dependence of all the angles after the perturbation; it gets pretty gory. Just find the new equilibrium position, demonstrate that the system is stable, and obtain the time evolution of the deviation $\delta\theta(t)$ from the new equilibrium given the initial impulses $\delta\dot{\theta}_0$ and $\delta\dot{\phi}_0$. Hints:

(a) A Lagrangian approach, like at the start of Section 8.10, is a convenient one for finding the conserved quantities and equations of motion.

(b) Note that $p_\phi = I_1\,\omega_P = L$ and $p_\psi = I_3\,\omega_3 = p_\phi\,\cos\theta$ are conserved quantities in the symmetric top problem (except possibly at the instant that the impulses are applied).

(c) On how to deal with the impulses – think about the way we dealt with such impulsive changes in velocity in connection with central force problems and when dealing with Green's functions for the SHO last term. Some quantities may undergo step changes, others must be continuous.

(d) Since the equilibrium position changes, don't try to demonstrate stability about the old equilibrium – you will get very frustrated!

[2]Mechanical gyroscopic stabilizers work by the use of two perpendicular spinning wheels whose shafts are mounted using bearings to the body to be stabilized near its center of mass. If a force tries to rotate the body, the bearings communicate the force to the wheels as a torque. Due to the angular momentum of the wheels, the angular acceleration that the destabilizing force can produce is reduced. Nowadays, though, most ships or other vehicles that require stabilization monitor a gyroscope to measure the motion relative to a fixed direction pointed at by the gyroscope spin axis and then apply countering motions to a fin or wheels. This is, for example, how a Segway works. Video cameras have begun to use respond to such motions by moving the the lens to counter the motion of the image rather than inertially stabilize the entire camera. Though you can purchase (large, massive) gyroscope stabilizers for cameras; see http://www.ken-lab.com/stabilizers.html.