Physics 106b/196b – Problem Set 11 – Due Feb 2, 2007

This problem set focuses on special relativity, Section 6 of the lecture notes. Problems 1 and 2 are for 106b students only, 3, 4, and 5 for 106b and 196b students, and 6 and 7 are for 196b students only.
There is an unobvious identity that you will find useful in Problems 2 and 4. If $\gamma^2 = \frac{1}{1-v^2}$, then it is straightforward to show $v^2 = 1 - \frac{1}{\gamma^2}$.

1. (106b) Hand and Finch 12-3
2. (106b) Hand and Finch 12-11
3. (106b/196b) Hand and Finch 12-8
4. (106b/196b) Hand and Finch 12-12
5. (106b/196b) Hand and Finch 12-23
6. (196b) Hand and Finch 12-18
7. (196b) In Section 12.17 of Hand and Finch, the Lorentz force is derived from the Lagrangian for a relativistic particle in an EM field (derived in Section 12.15) via the standard Lagrangian procedure. But that procedure puts time on special footing. It turns out that the result Equation 12.100 can be written in a form, Equation 12.101, that is manifestly Lorentz-covariant, meaning that it is obvious how to apply Equation 12.101 in different Lorentz frames. Can you develop a manifestly covariant Euler-Lagrange procedure to go from the Lorentz-invariant action

$$\mathcal{S} = \int d\tau \, (-m - q \, u_\mu \, A^\mu)$$

to the manifestly covariant Lorentz force equation, Equation 12.101? Refer to Hand and Finch Problem 5-6 for some guidance. The 4-vector potential A^μ is first discussed in Section 12.11 and you may use the fact (remarkably, not stated in Hand and Finch) that the field strength tensor $F^{\mu\nu}$ shown in Equation 12.76 is given by

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad \text{with} \qquad \partial^\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla} \right)$$

You will likely need to refer to sections of Hand and Finch that were not assigned reading, and to Section 6.1.3 of the lecture notes, to do this problem. You should set $c = 1$ for simplicity.