Neutrino Oscillations

Gary Cheng
Physics 135c
6/1/07

Introduction: Theory

- Neutrinos have mass eigenstates v_1, v_2, v_3 that are superpositions of the flavor eigenstates v_e, v_μ, v_τ , the quantum states in which neutrinos are produced.

- The difference between the mass eigenstates and the flavor eigenstates of neutrinos is what causes neutrino oscillations.

Introduction: Theory 2

- The mass eigenstates v_1, v_2, v_3 are the neutrino eigenstates of vacuum space.

- The phase change acquired during the time evolution of the mass eigenstates v_1, v_2, v_3, whose components add up to make the detectable flavor eigenstates v_e, v_μ, v_τ gives the finite probability that a neutrino emitted as one flavor eigenstate is later detected as another flavor eigenstate after traveling through space.

Introduction: Theory 3

- Two neutrino example:

$$U = \begin{array}{c} \\ \nu_\alpha \\ \nu_\beta \end{array} \begin{array}{c} \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \end{array}$$

Introduction: Theory 4

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

$$|\nu_\alpha(L)\rangle \approx \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L} |\nu_i\rangle$$

$$|\nu_\alpha(L)\rangle \approx \sum_\beta \left[\sum_i U_{\alpha i} e^{-i(m_i^2/2E)L} U_{\beta i} \right] |\nu_\beta\rangle$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \, \sin^2 [1.27 \, \Delta m^2 (L/E)]$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \, \sin^2 [1.27 \, \Delta m^2 (L/E)]$$

Neutrino Mixing Matrix (MNS Matrix)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \overbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{\text{Solar, Reactor}} \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}^{\text{Atmospheric, Accelerator}}$$

$$\times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ \ s_{ij} \equiv \sin\theta_{ij},\ \{\delta, \alpha_1, \alpha_2\} \equiv \text{CP - Violating Phases}$$

Neutrino Mixing Matrix (MNS Matrix)

$$U = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \end{array}$$

$$\times \operatorname{diag}(e^{i\alpha_1/2},\ e^{i\,\alpha_2/2},\ 1)$$

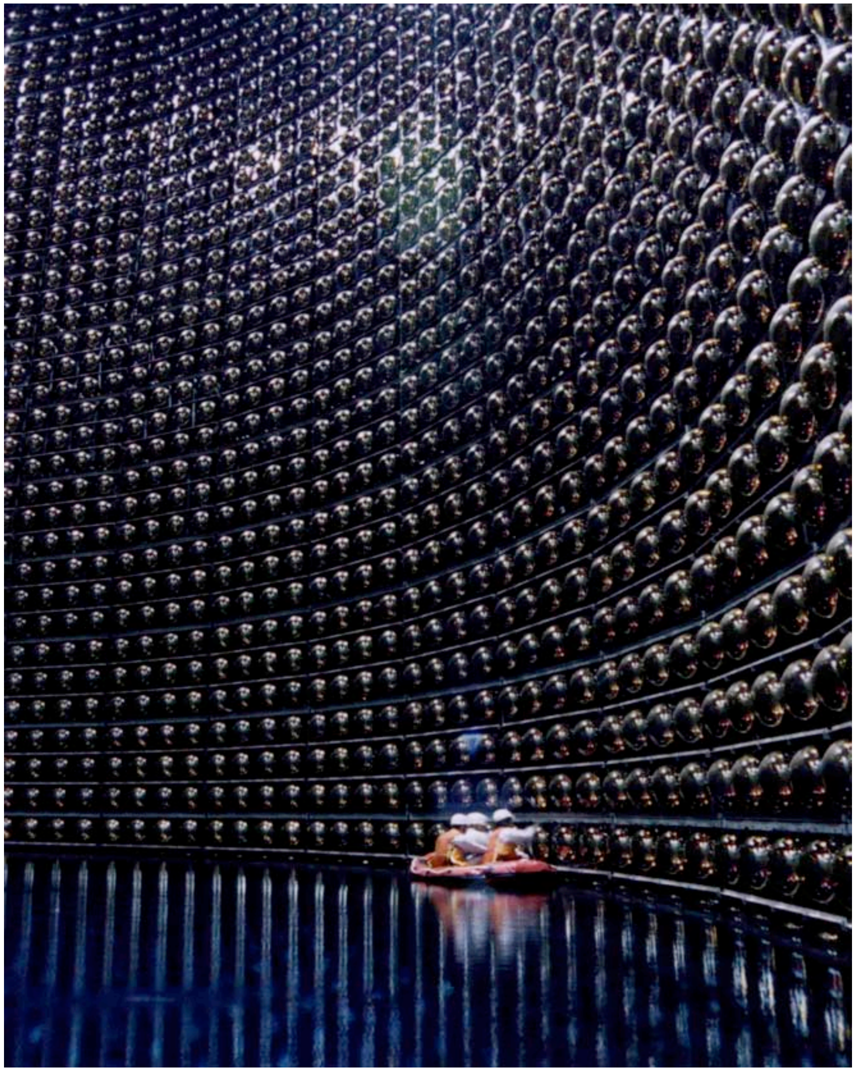
Origin of Neutrino Oscillations

- Ray Davis's Homestake Experiment observed a deficit in the number of solar v_e neutrinos reaching the Earth as predicted by the standard solar model. Roughly 1/3 of the expected number of neutrinos is detected.

- The experiment consisted of a large tank of liquid C_2Cl_4 placed underground with the reaction:

$$v_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$$

Description of Super-Kamiokande



- Consists of a 50,000 ton cylindrical tank of ultra-pure water surrounded by 11,242 inward facing photomultipler tubes (PMTs).
- Placed 1000 meters within the Kamioka Mine to reduce background.

Confirmation of Neutrino Deficit

- Super-Kamiokande confirms the deficit in solar v_e neutrinos reaching the Earth. Only about 35% of the expected number of neutrinos is detected.

- v_e either scatters an electron or interacts with H to produce a positron. The energetic electron or positron emits Cherenkov radiation as it travels through the water and the radiation is detected by the photomultipliers.

Solar Neutrinos

- The electron density in the sun as the v_e travels outward from the center of the sun affects the mixing angle. This is known as the matter effect.

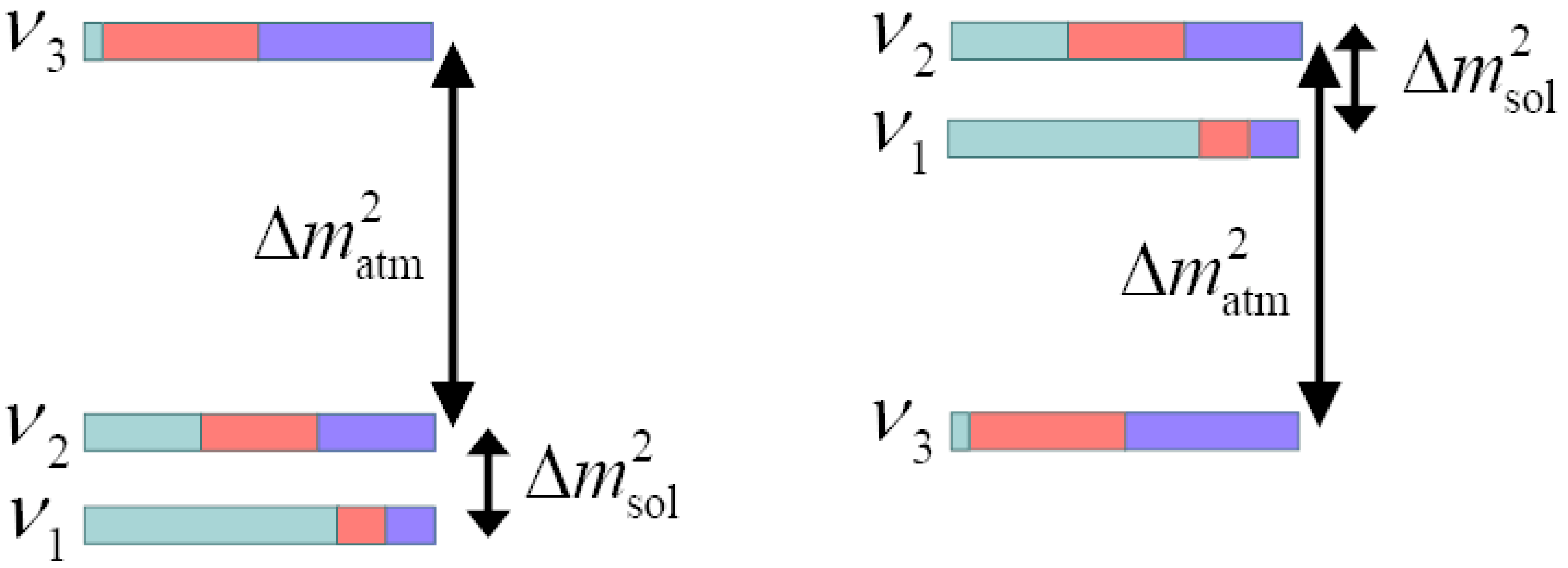
$$
\begin{aligned}
\mathcal{H} &= \mathcal{H}_V + \mathcal{H}_M(r) \\
&= \frac{\Delta m_\odot^2}{4E} \begin{bmatrix} -\cos 2\theta_\odot & \sin 2\theta_\odot \\ \sin 2\theta_\odot & \cos 2\theta_\odot \end{bmatrix} + \begin{bmatrix} V(r) & 0 \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

Result of Solar Neutrino Matter Effect

- From neutrino oscillations in vacuum, it is impossible to distinguish the mixing angle θ from θ' = π/2 – θ.

- But because of the *V(r)* term in the Hamiltonian as the solar ν_e travels through the electron density in the sun, it is possible to distinguish θ from θ'.

Mass Hierarchy of Neutrinos



"Normal" Hierarchy "Inverted" Hierarchy

ν_e 🟦 ν_μ 🟥 ν_τ 🟪

Current Accepted Values of Mass Differences and Mixing Angles

- Atmospheric (and Accelerator):

$$1.9 \times 10^{-3}\,\mathrm{eV}^2 < \Delta m^2_{\mathrm{atm}} < 3.0 \times 10^{-3}\,\mathrm{eV}^2$$

$$\sin^2 2\theta_{\mathrm{atm}} > 0.90$$

- Solar (and Reactor):

$$\Delta m^2_{\odot} = (8.0^{+0.6}_{-0.4}) \times 10^{-5}\,\mathrm{eV}^2$$

$$\theta_{\odot} = (33.9^{+2.4}_{-2.2})^{\circ}$$

Determining the Parameters of Neutrino Oscillations

- From the neutrino oscillation probability formula, the oscillations depend on three critical parameters: Δm^2 (difference between the masses of the neutrinos squared), L (distance from emitted neutrino), E (energy of neutrino).

Separation of the Mixing Angles

- The separation of the three mixing angles is due to the following factors:
  - θ_{13} is small compared to θ_{23} and, to a first approximation, can be neglected.
  - The two order of magnitude difference between Δm_{12} and Δm_{23} ($\Delta m_{23} \approx \Delta m_{13}$) allows the neutrino oscillations to “separate” because the wavelength of oscillations are on entirely different length scales for a given energy of the neutrino.

Solar Neutrino Approximation

- The baseline for solar neutrinos is the distance from the sun to the earth (about 150 million km).
- The solar v_e energy is in the few MeV range.
- The oscillations detected are of the mixing angle θ_{12} and $\theta_{solar} \approx \theta_{12}$ (the length scale of θ_{13} and θ_{23} are too short to make a significant contribution).

Atmospheric Neutrino Approximation

- The baseline for atmospheric neutrinos is the diameter of earth (about 12,500 km).

- The atmospheric ν_{τ} energy is in the hundreds of MeV to few TeV range.

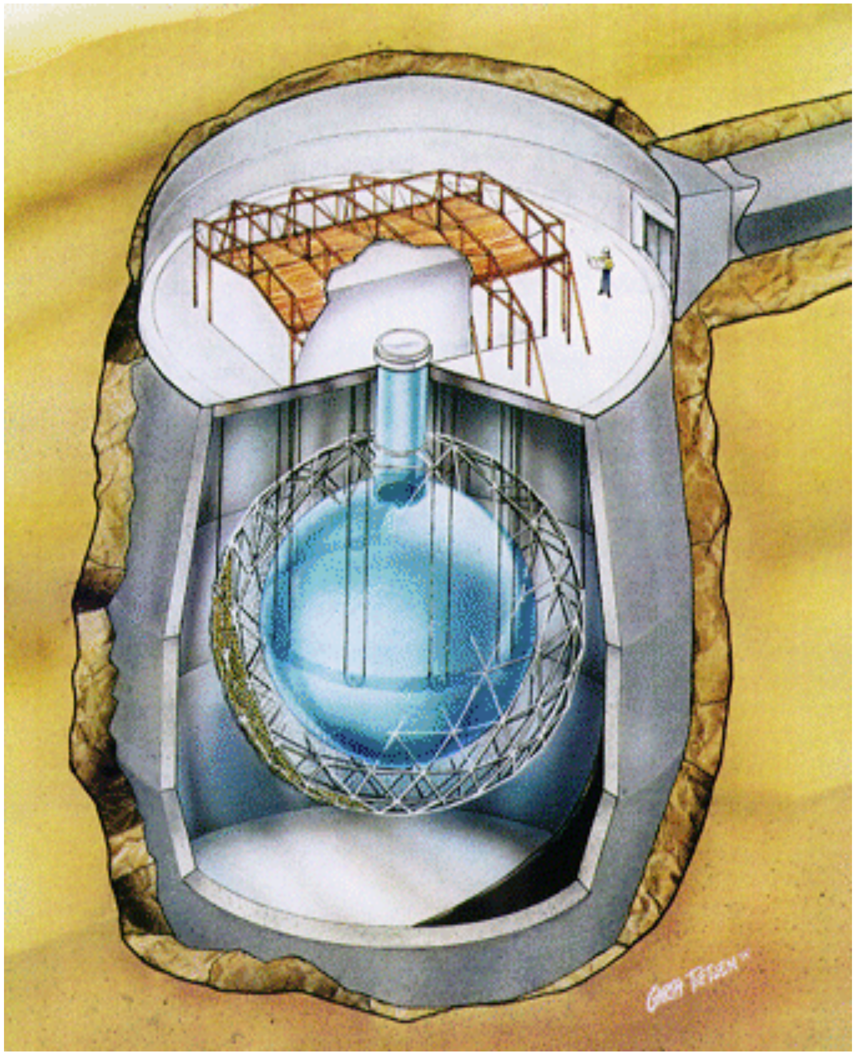
- The oscillations detected are of the mixing angle θ_{23} and $\theta_{atm} \approx \theta_{23}$ (the length scale of θ_{12} is too long to make a significant contribution and θ_{13} is small and neglected).

Criteria for Neutrino Oscillation Experiments

- All current neutrino oscillation experiments are designed with $\Delta m^2(L/E)$ in mind.

- For the given θ to be measured, the distance of the detector from neutrino production source (L) and the energy of the neutrino (E) are chosen so that $\Delta m^2(L/E)$ is on the order of magnitude of π.

SNO (Solar Neutrinos)

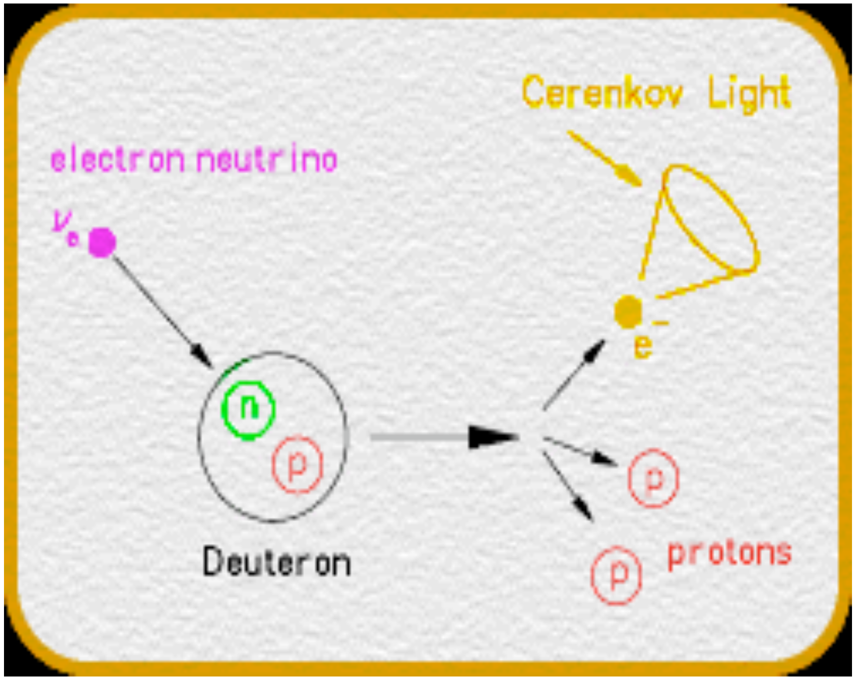


- 1000 tons of heavy water (D_2O) in a 850 cm spherical vessel surrounded by approximate inward facing 9600 PMTs.
- Located 2 km underground in a mine in Ontario, Canada.

SNO (Sudbury Neutrino Observatory)

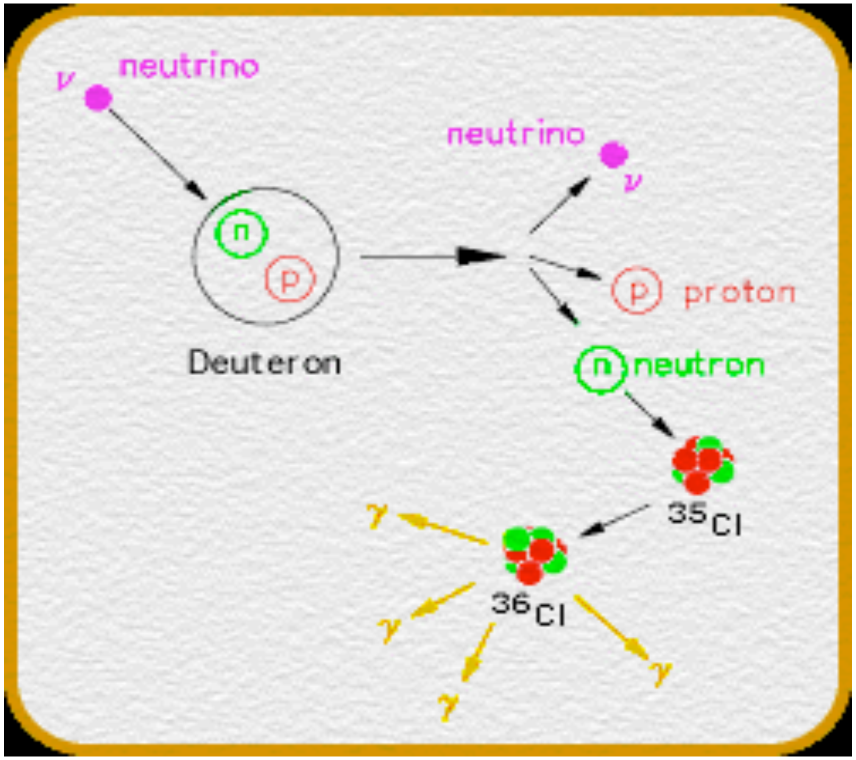
- The SNO Experiment could detect all three flavors of neutrinos v_e, v_{μ}, v_{τ} coming from the sun through three possible interactions:
  - Charged current reaction
  - Neutral current reaction
  - Electron scattering

Charged Current Reaction



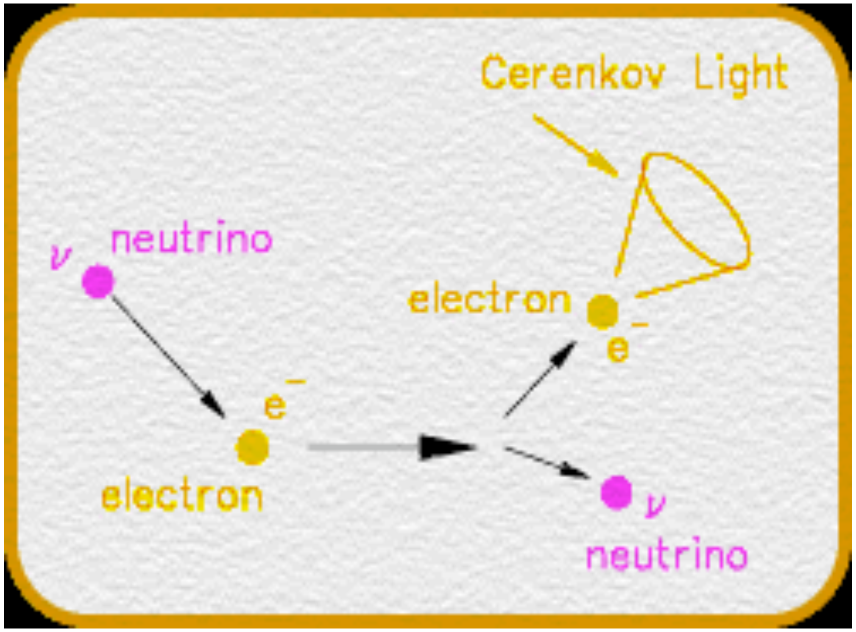
- $\nu_e + d \rightarrow p + p + e^-$
- Only sensitive to ν_e.

Neutral Current Reaction



- $\nu_i + d \rightarrow n + p + e^- + \nu_i$
- Sensitive to all three neutrino flavors with equal cross-sections.

Electron Scattering

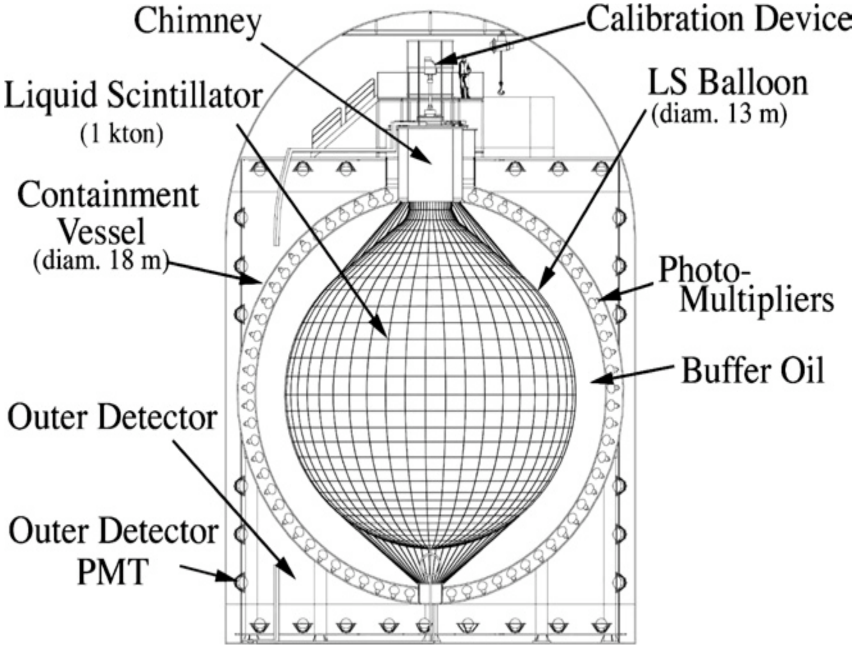


- $v_i + e^- \rightarrow v_i + e^-$
- Sensitive to all three neutrino flavors, but v_e sensitivity dominates by a factor of 6.

SNO Results

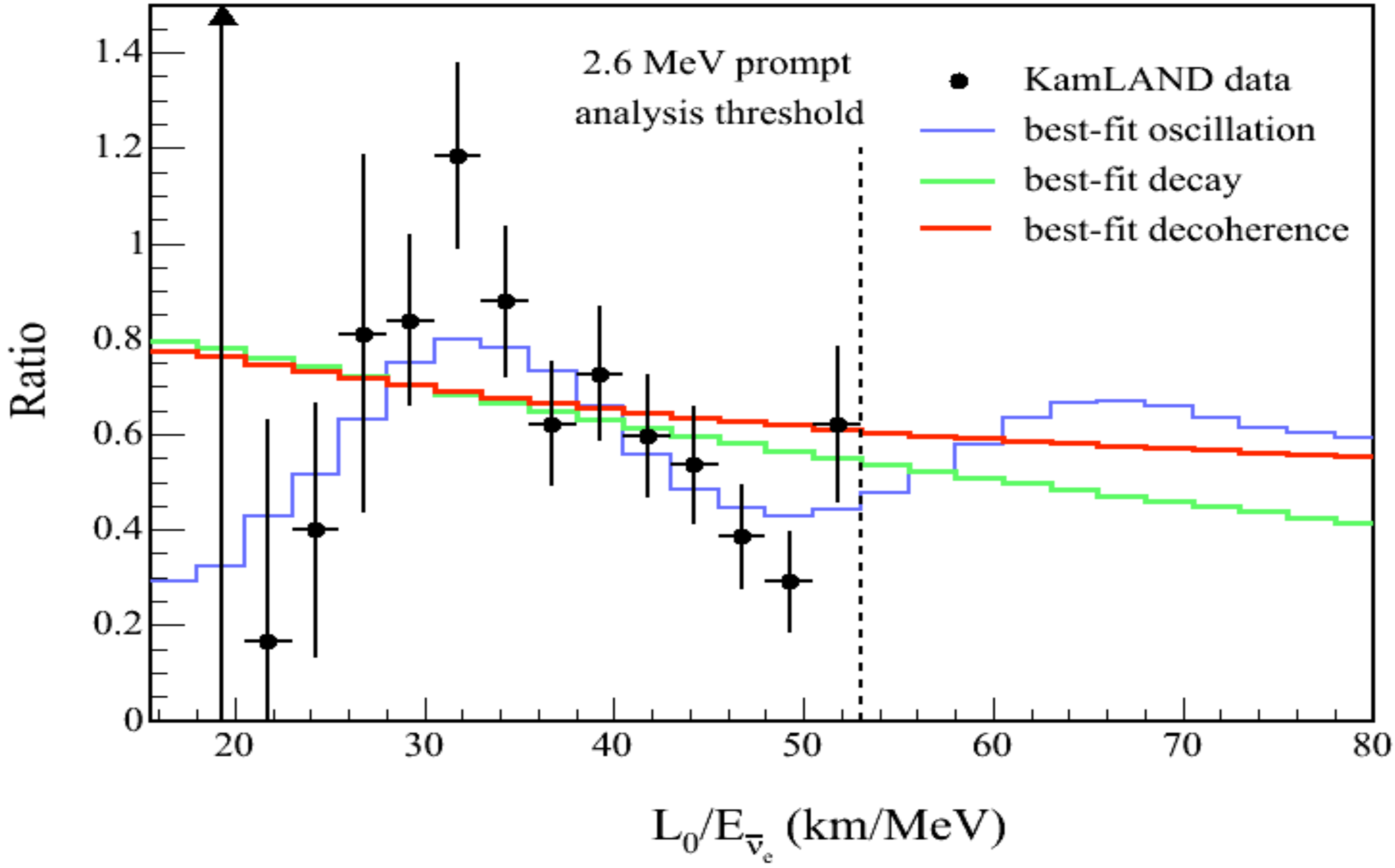
$$\frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.340 \pm 0.023\,(\mathrm{stat})\,{}^{+0.029}_{-0.031}\,(\mathrm{syst})$$

KamLAND (Reactor Anti-neutrinos)

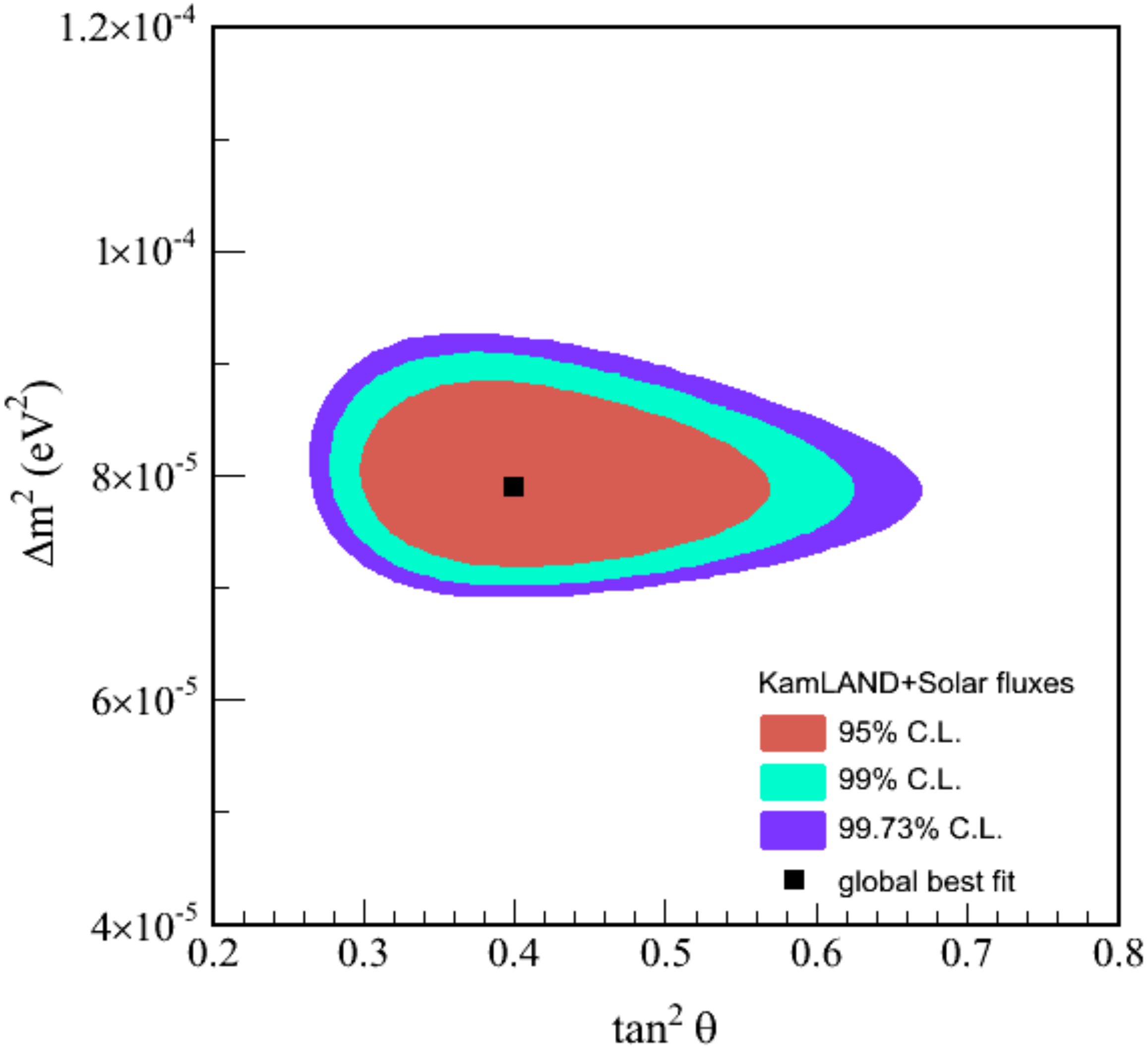


- KamLAND (Kamioka Liquid scintillator Anti-Neutrino Detector) detects antineutrinos from dozens of Japanese nuclear reactors, mostly located 150-200 km away.
- 1000 tons of 80% dodecane and 20% pseudocumene in a roughly 13 m diameter nylon/EVOH balloon surrounded by 1,879 PMTs.

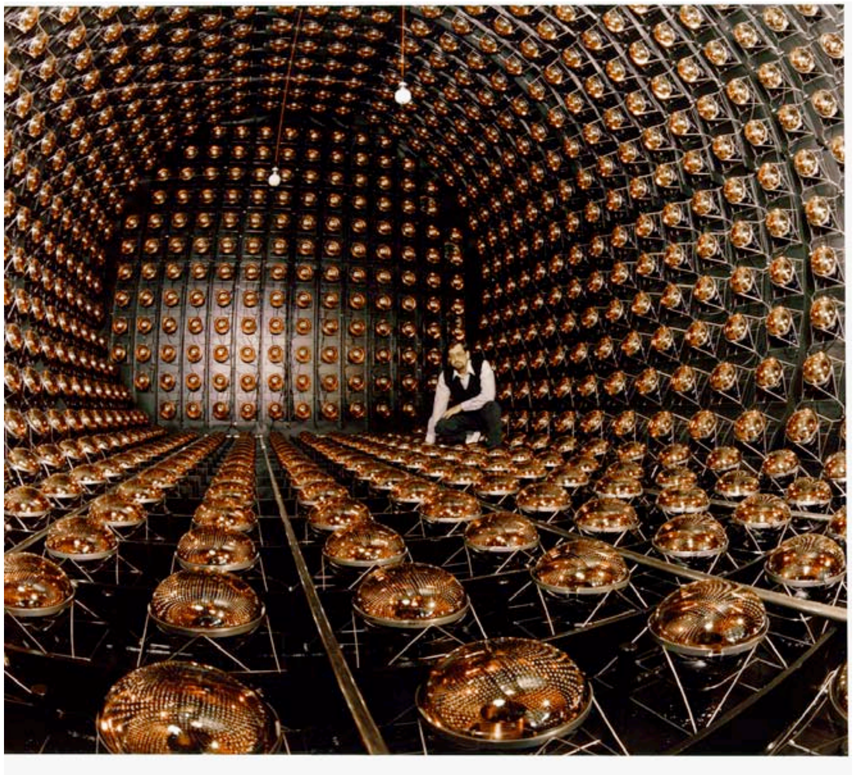
KamLAND Results



Solar Exp. and KamLAND Data



LSND (Accelerator Neutrinos)



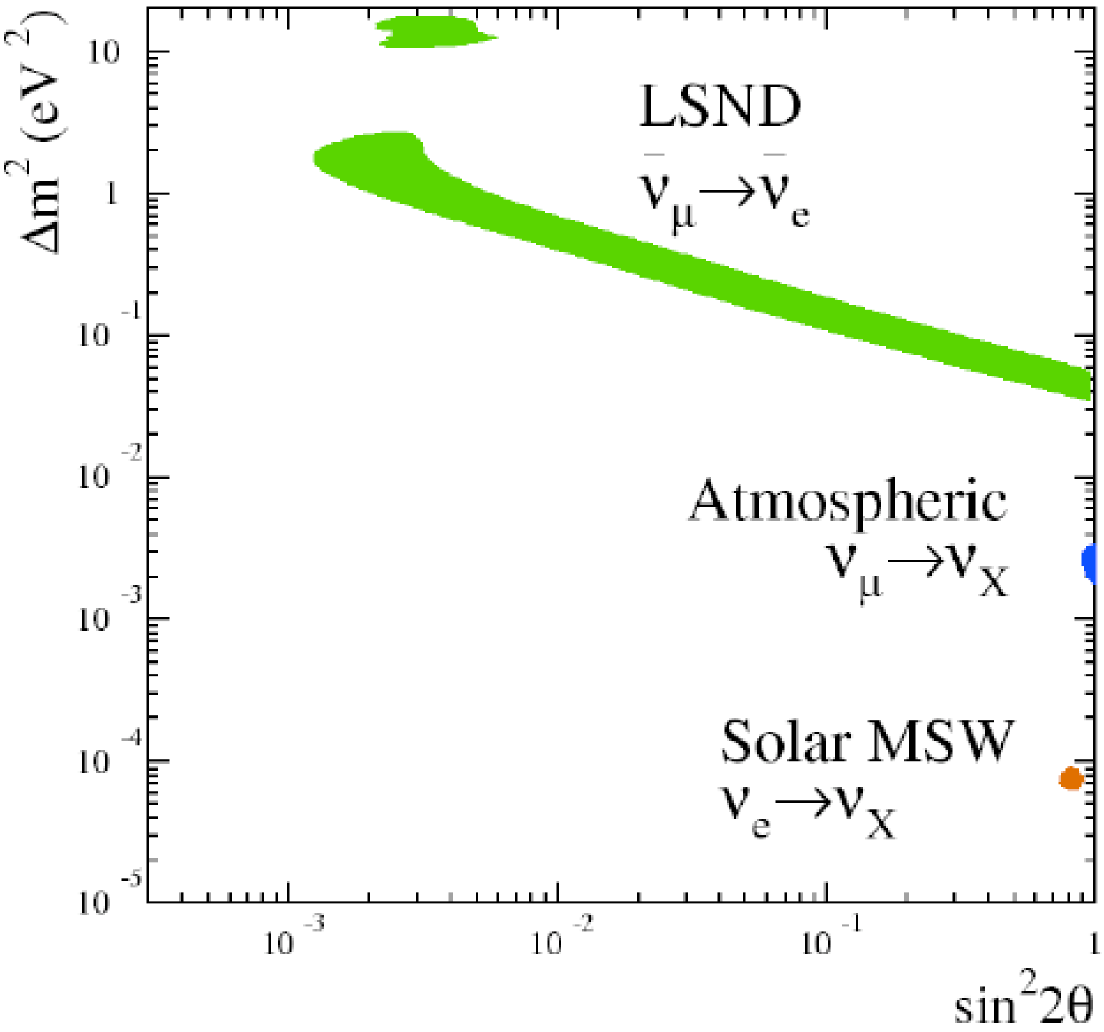
- LSND (Liquid Scintillator Neutrino Detector) detects the excess of electron antineutrinos above background oscillating from muon antineutrinos.
- Muon antineutrinos are created from the decay of at rest μ+.

$$\mu^+ \to e^+ \nu_e \overline{\nu}_\mu$$

LSND Experiment

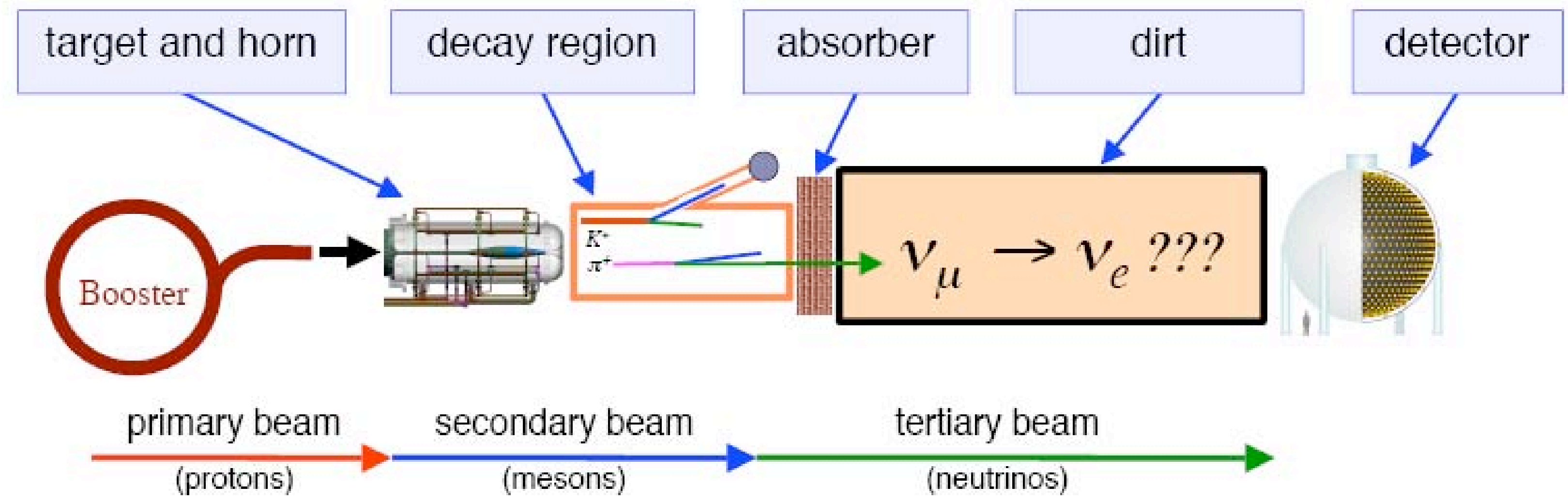
- The detector consists of a 167-ton tank of mineral oil surrounded by 1220 PMTs.
- The baseline for the muon antineutrino oscillation is roughly 30 m.
- LSND results indicated a fourth sterile neutrino which was refuted by MiniBoone in 2007.

LSND Anomalous Result



$\Delta m^2_{sol} + \Delta m^2_{atm} \neq \Delta m^2_{LSND}$

MiniBoone



- Detector is 40 m diameter sphere containing 800 tons of mineral oil surrounded by 1520 detectors.
- MiniBoone detects excess ν_e in a ν_μ beam with a neutrino oscillation baseline of 450m.

MiniBoone Result 1

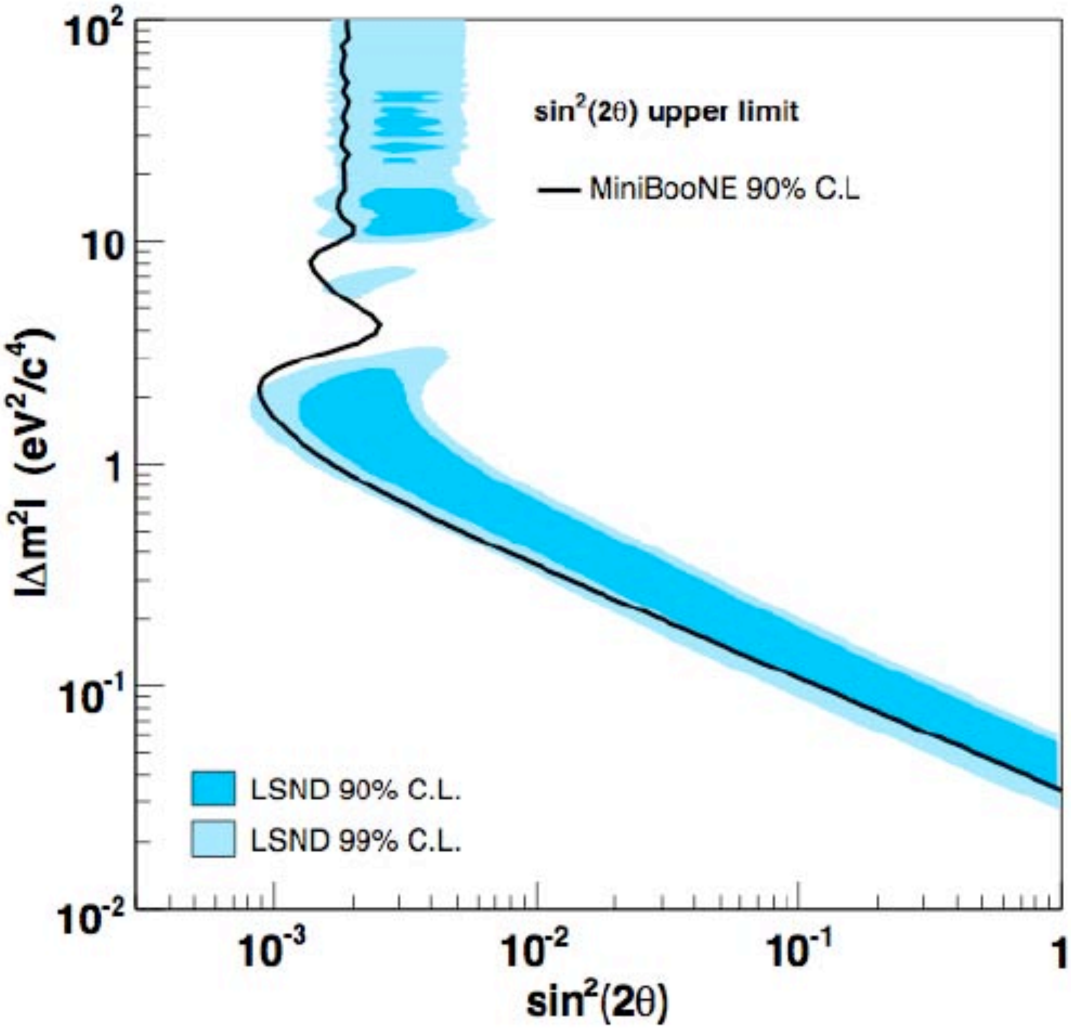
The Track-based $\nu_\mu \rightarrow \nu_e$ Appearance-only Result:

Counting Experiment: $475 < E_\nu^{QE} < 1250$ MeV

data: 380 events
expectation: 358 ±19 (stat) ± 35 (sys) events

significance:
0.55 σ

MiniBoone Result 2



Future Experiments in Neutrino Oscillations

- Neutrino oscillation experiments (Double Chooz, Daya Bay, etc.) are underway to measure the small θ_{13} angle.

- Both Double Chooz and Daya Bay will watch for the disappearance anti-v_e from nuclear power reactors by comparing flux and energy spectrum between detectors located close and far from the reactors.